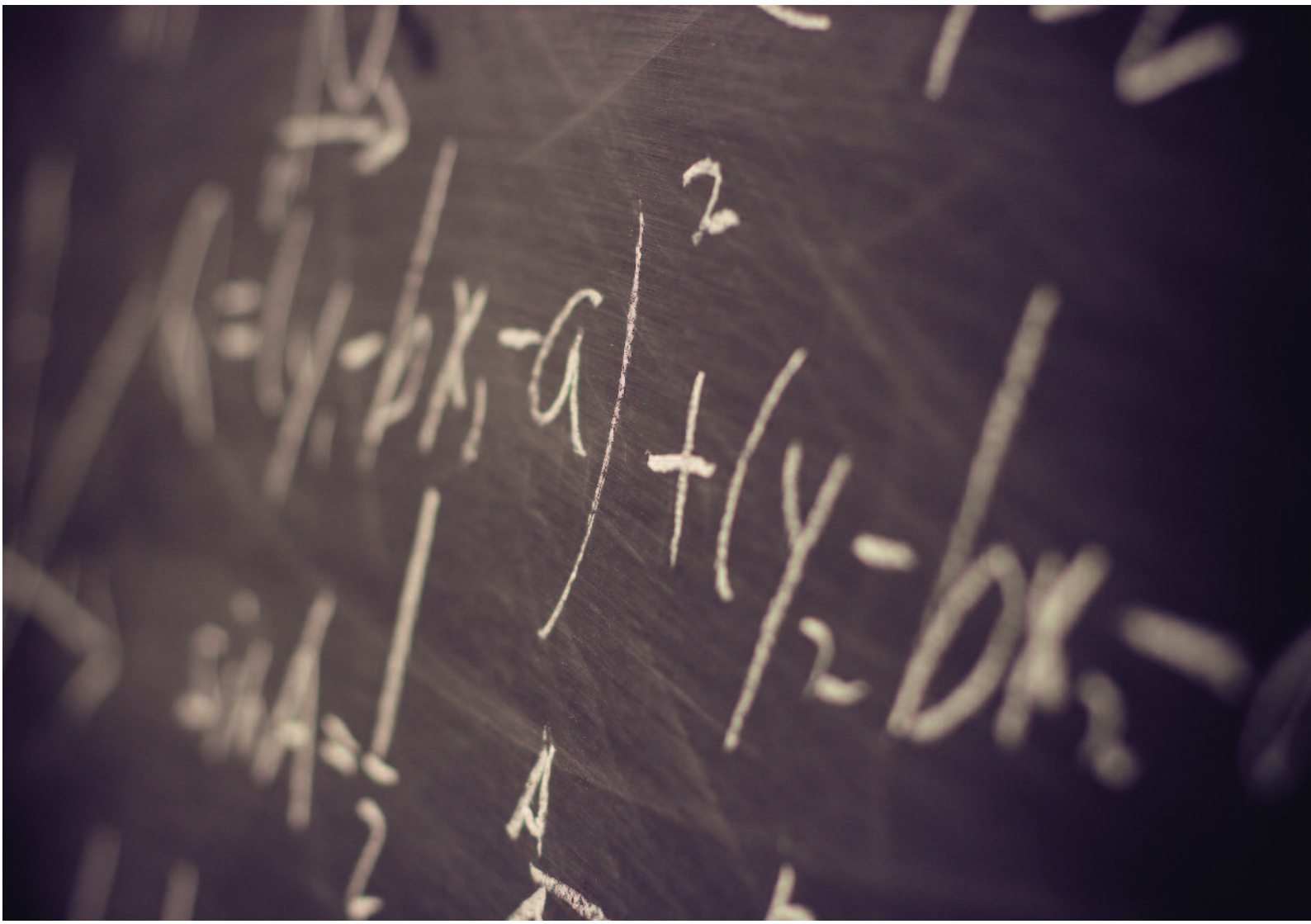
Mathematics with industry: driving innovation

Annual Report 2025

ECMI EUROPEAN CONSORTIUM FOR **MATHEMATICS** IN **INDUSTRY**

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Editorial

Dear Readers,

Once again, welcome to all, whether ECMI members or people who just take pleasure in reading industrial mathematics news. In this report, as well as the usual wide variety of activities, reflecting the broad range of interests of ECMI members, there are multiple contributions from both Fraunhofer ITWM and Kaunas University – a special thanks to those contributors.

In last year's Editorial I bemoaned the reputational damage that mathematics in general has suffered, through fraudulent publications and manipulation of citations (of course mathematics is not the only victim). This led to Mathematics being excluded from the Clarivate Highly Cited Researcher list. In response to this the IMU and ICIAM joined forces to publish two key works addressing the issue "How to fight fraudulent publishing in the mathematical sciences: Joint recommendations of the IMU and the ICIAM" [1] and "Fraudulent publishing in the mathematical sciences" [2]. In the meantime Clarivate brought in changes designed to eliminate certain groups, for example by removing highly cited papers authored by researchers previously excluded for integrity issues; with excessive self- or group-citation and publication rates; as well as authors with an anomalous ratio of reviews to articles. As a result Mathematics has now returned to the list. Let's try to keep it in there!

On a related note, in the Activities section we have an article by Andras Batkai concerning the ethical dimensions of mathematical modeling and the consequences of our decisions, through conscious or unconscious bias. A comment that struck me, and that I am trying to introduce to my own work, is to also discuss errors and approaches that have failed - we can learn as much from errors as successes. The first version of the book that inspired the article will be published in German [3] but an English translation is intended in the near future. If you don't speak German, I recommend buying the book and learning.

I would like to take this opportunity to record the passing of Prof. Sarah Mitchell in May 2025. Sarah was my post-doc In Cape Town before moving to the Mathematics Applications Consortium for Science and Industry (MACSI) at the University of Limerick, where she became

Head of Department from 2016-2020. Sarah was dedicated to research, teaching and also to advancing gender equality in STEM subjects, particularly with high school students. She was an active member of ECMI, representing Ireland on the Council until 2020, when COVID and illness restricted her travel. Our last council meeting was the infamous Milan meeting of 2019, where the champagne flowed like water until the treasurer found out the price! Our collaboration included research on football motion, moving boundaries, heat transfer integral methods and spontaneous combustion. Many topics arose through a commitment to supporting applied mathematics in South Africa. Her popularity could be measured through the continual flow of colleagues and friends throughout her illness. I would recommend visiting MACSI, even if it is just for their atmosphere of support and collaboration.

I worked with Sarah on more topics, in more countries and for longer than any other collaborator and will always miss her contributions. Whether you knew her or not, please raise of glass of appropriate liquid and think of her!



Finally, I would like to thank everyone who has contributed to this issue and encourage ECMI members (and even non-members) to propose topics for next year's report. I would also welcome new ideas and suggestions which may not quite fit in with the current categories. We must evolve to stay relevant!

Tim Myers,
Centre de Recerca Matemàtica
July 2026

"No human investigation can be called real science if it cannot be demonstrated mathematically."
– Leonardo da Vinci (1651)

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Welcome from the President

Dear Colleagues,

The year 2025 has been marked by a wide range of activities across our ECMI community, once again demonstrating our shared commitment to advancing Industrial Mathematics throughout Europe and beyond.

Our Special Interest Groups continued to play a central role in fostering scientific collaboration and innovation. Many funded projects, doctoral and postdoctoral research activities have been carried out in ECMI centres, covering areas such as Sustainable Energies, Mathematics for Digital Factories, Modelling, Simulation and Optimization in Electrical Engineering, Computational Finance and Energy Markets, Mathematics for Big Data and Artificial Intelligence, and Computational Physiology and Mathematical Epidemiology. The many contributions published on the ECMI blog throughout the year testify to the vitality and diversity of these activities.

Education remains at the heart of ECMI's mission. The ECMI Modelling Week, organized in Kaunas on June 29 to July 5, 2025 once again provided students with a unique opportunity to work on challenging industrial problems in an international environment, while the ECMI Student Competition continued to attract talented young mathematicians from across Europe. These initiatives are essential in preparing the next generation of industrial mathematicians and strengthening the connection between academia and industry.

A particularly encouraging development during 2025 has been the continued growth of our network. Two new centres, Gdańsk University of Technology in Poland and KU Leuven in

Belgium, joined ECMI, further strengthening our educational and research capacity. We were also delighted to welcome Belgium among the countries represented in the ECMI Council. In addition, universities from North Macedonia, Greece, and Georgia have expressed interest in joining ECMI, opening exciting perspectives for further extending our geographical reach and reinforcing our European dimension.

Throughout the year, European Study Groups with Industry, Industrial Days, workshops, and specialized meetings continued to provide valuable opportunities for interaction between mathematicians and industrial partners. The “Mathematics in Industry Reports” repository, developed in collaboration with Cambridge University Press, has been up to now an important vehicle for disseminating the outcomes of these activities.

ECMI also continued its active engagement with partner organizations. Our long-standing collaborations with the European Mathematical Society through the Committee for Applied and Interdisciplinary Relations, ICIAM, EU-MATHS-IN, and the Asia-Pacific Consortium for Mathematics in Industry have further strengthened. In addition, following our membership in the Initiative for Science in Europe (ISE), ECMI has become increasingly involved in discussions concerning the future European research landscape, contributing to efforts aimed at ensuring adequate support and funding for scientific research within the forthcoming FP10 framework.

Preparations are almost finalized for the 23rd ECMI Conference, which will take place in Kaunas, Lithuania, from June 29 to July 3, 2026. We are looking forward to welcoming the entire ECMI community to what promises to be another outstanding scientific event, offering opportunities to strengthen collaborations between academia and industry and to promote innovation through mathematics.

The Journal of Mathematics in Industry continues to grow in visibility and impact, reflecting the increasing relevance of industrial mathematics in addressing contemporary scientific and technological challenges. I would like to thank the editorial team, reviewers, and authors for their continued dedication and contributions.

Starting from 2026 we have also seen transitions in some of the ECMI leadership roles. I would like to express my sincere gratitude to all colleagues who have served ECMI in various capacities and whose commitment has contributed significantly to the development of our consortium over the years.

Finally, as always, I encourage all of you to continue following our blog, supporting ECMI initiatives, and sharing your ideas and proposals. We warmly welcome new members, new centres, and new perspectives as we continue to build a strong and diverse European network devoted to industrial mathematics.

Thank you for being part of ECMI and for your enthusiastic participation in all our activities.

Alessandra Micheletti, Università degli Studi di Milano, June 2026.

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Activities and Initiatives

SUM4Products: A new UK network for mathematics in formulation engineering

Formulated products are everywhere; they are foods, drinks, soaps, creams, lotions, cleaning products, medicines, and more. Despite the scope and maturity of the formulated products sector, it still faces multiple overarching challenges. Societal changes have prompted the need for greener, more sustainable, and personalised products. Supply chain issues and regulatory changes motivate the need for ingredient substitutions. The challenge is then determining, without costly trial and error, what substitutes will retain the taste, texture, or scent of the original product.



In June 2025, the Scale-up Mathematics for Formulated Products (SUM4Products) Network was created to address current critical challenges in the formulations sector. The UK-based network is supported by a £2.8M grant awarded by the Engineering and Physical Sciences Research Council. The network is led by Rosemary Dyson (U Birmingham), co-led by Matthew Hennessy (U Bristol), Halim Kusumaatmaja (U Edinburgh), Estefania Lopez-Quiroga (U Birmingham), and Helen Wilson (UCL), and managed by Sophie Cheng (U Birmingham). The network will connect academics from multiple disciplines, including engineering, physics, and mathematics, with industrial

partners from companies of all sizes, from micro-SMEs to multi-nationals.

At the heart of the SUM4Products vision is a mathematics-driven approach to problem solving. By drawing on areas such as fluid and solid mechanics, heat and mass transfer, and multi-scale methods, the network aims to develop predictive modelling capabilities that will unlock innovation across the sector. A key goal is to open the doors to engineering the taste, feel, and texture of formulated products by varying individual ingredients or processing parameters, with limited need for trial-and-error experiments. The network is currently focussing on three broad research themes identified through consultation with industry partners:

1. Developing scalable methods for heat and mass transfer and reactions in multiphase systems. Most formulated products are complex multiphase, multicomponent mixtures whose lifecycle is driven by heat and mass transfer processes coupled to chemical reactions. These processes occur across disparate length and time scales as the formulation goes from production to shelf and finally to consumer use.
2. Developing multi-scale models for materials with transforming microstructure. Across the sector, there is critical need to predict how material microstructure manifests in macroscopic response and function. Linking microstructure to macrostructure requires the development of multi-scale mathematical and computational approaches, potentially using homogenisation or machine-learning methods. This will lead to a better capability to predict the impact of replacing one ingredient with another, e.g. to obtain optimal behaviour, to minimise waste during processing, or in response to a regulatory change or change in supplier.
3. Developing novel mathematical methods to quantify sensory perception. What matters most to industry partners is the sensory perception of their customers. A chocolate manufacturer will want to know if their chocolate tastes bitter; a deodorant manufacturer will want to know if their deodorant feels wet or sticky. The overarching challenge is to complement or eventually replace subjective measures, such as consumer group surveys, with quantitative measures linking microstructure transformation and heat and mass transfer to perception.

The SUM4Products network acknowledges financial support from the UK Engineering and Physical Sciences Research Council (grant number UKRI951).

To make this all possible, SUM4Products is organising a series of annual events and funding calls. The events are designed to provide opportunities for building industry collaborations as well as identifying and working on industry-driven challenges. Keeping with the spirit of ECMI, an annual study group, specific to formulation engineering, will be organised. The SUM4Products Short Research Project Scheme will provide funding (up to 3 months FTE) for early-career researchers to continue work on study group problems or start new research projects with industry collaborators. In partnership with the UK Knowledge Exchange Hub for Mathematical Sciences, triaging workshops will be organised for scoping problems that could be developed into short research projects to be funded by the network.

To boost the research capacity of the academic members of the network, SUM4Products is funding four full-time innovation research associates (InRAs). InRAs are post-doc level roles where the focus is on completing several short-term impact-driven research projects with industry over the duration of the post. Any faculty-level academic member of the SUM4Products network can bid on InRA time through the Short Research Project Scheme, providing an opportunity to bring a dedicated researcher onto their formulations-related project. In addition to working on specific industry-led research projects, the InRAs will also work on cross-cutting research challenges that have broad impact across the formulations sector.

As the first birthday of the network approaches, it seems appropriate to look back on the events that have taken place so far. A network launch event held in Birmingham in November brought together 80 academics and 13 industry partners, leading to one short research project funded by the network. The first SUM4Products study group took place in April 2026 at the University of Birmingham. Representatives from Mondelez International, PepsiCo, and Afton Chemical proposed problems to about 30 academic participants. The problems spanned multi-scale modelling of cocoa bean fermentation, accelerating molecular dynamics calculations, and modelling the rheology of oil-water emulsions. A carefully curated social programme was organised, which culminated in a karaoke session with an eclectic soundtrack covering Pet Shop Boys, Kate Bush, and, of course, Black Sabbath.

Looking ahead, the 2026 annual meeting for the SUM4Products network will be taking place in Edinburgh on Sept 18-19. The next SUM4Products study group on formulation engineering will take place at the Isaac Newton Institute in Cambridge on 19-23 April 2027. Participation at both events is free, and some travel funding is available.

Membership of the network is open to all, and we encourage anyone that is keen to work at the interface of applied maths, physics, and engineering to get involved. To learn more about the network, sign up to be an academic member, or join as an industry partner, you can visit our website at https://sum4products.co.uk/. You can also stay updated on SUM4Product activities by signing up to our mailing list.

Matthew Hennessy

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The Hidden Weight of Numbers: Ethical Imperatives in Industrial Mathematics

Mathematical modeling in industry is frequently perceived as an objective, neutral process governed purely by logic and empirical data. However, the application of mathematics to real-world industrial, financial, and societal systems inevitably involves implicit value judgments. This article explores the ethical dimensions of mathematical modeling for applied mathematicians. By examining historical and contemporary case studies ranging from catastrophic engineering failures to systematic biases in financial algorithms and the manipulation of academic metrics, we argue that every phase of modeling, especially the act of abstraction, carries profound ethical weight and real-world consequences.

Introduction: The Myth of Neutrality

Mathematics has traditionally been viewed as a sanctuary of absolute truth. Governed by rigorous proofs and universal axioms, it is widely considered the most objective of all scientific disciplines. In pure mathematics, a statement is either true or false, entirely independent of the mathematician's personal beliefs, cultural background, or societal norms.

However, the moment mathematics steps out of this abstract realm and is applied to the physical and social world—the daily domain of the industrial and applied mathematician—this inherent neutrality vanishes. When we transition from pure mathematics to mathematical modeling, we cross a critical threshold. We are no longer merely discovering universal truths; we are actively making choices about how to represent reality.

In industrial applications, every stage of the modeling process requires decisions that are fundamentally normative. Deciding which variables to include and which to ignore, selecting a specific level of precision, choosing between truncation and rounding, or determining the acceptable margin of error—these are not just technical steps, but value judgments. The very act of abstraction, which is necessary to make any complex real-world problem mathematically tractable, requires us to declare certain aspects of reality "irrelevant."

For the applied mathematician working in engineering, finance, or policy, these "irrelevant" details can have catastrophic real-world consequences if misjudged. A model is never a perfect mirror of reality; it is a purposeful simplification. When industry leaders, politicians, or the public place blind trust in the "objective" output of an algorithm or a computational simulation, they often fail to see the human assumptions embedded within the code.

In this article, we aim to demonstrate that mathematical models are not simply passive tools; they are active agents that shape infrastructure, economies, and human behavior. By examining cases where seemingly trivial numerical choices led to massive industrial failures or systemic biases, we highlight why a rigorous ethical framework is just as vital to industrial mathematics as numerical stability and computational efficiency.

The Catastrophic Cost of “Trivial” Errors

Within the realm of pure mathematics, a calculation is a closed, abstract system where $2 + 2$ equals 4 without exception. Errors, when they occur, are generally recognized as mere technical oversights. However, when mathematical models are applied to complex industrial engineering, the character of an error fundamentally changes. In the physical world, a numerical mistake ceases to be trivial the moment it triggers irreversible, real-world actions. The ethical duty of an applied mathematician therefore extends far beyond mere internal mathematical correctness; it demands a rigorous, context-aware duty of care.

Consider the Spanish submarine project S-80. In the early 2000s, an ambitious initiative to develop a new class of diesel-electric submarines was launched. However, during the design phase, a seemingly trivial error occurred: a magnitude error regarding material weights, likely due to a misplaced decimal point or a faulty unit conversion. Because this foundational calculation was not subjected to independent plausibility checks, the error propagated through the entire development cycle unnoticed.

The physical consequences were discovered only shortly before the planned delivery: the submarine was between 75 and 100 tons too heavy, effectively lacking the necessary reserve buoyancy to safely return to the surface after diving. To save the project, the submarine’s hull had to be lengthened by ten meters to generate more volume and buoyancy. This structural change, however, led to a cascading infrastructure failure: the newly lengthened submarine no longer fit into the existing docks at the naval base in Cartagena, necessitating massive, costly dredging and expansion of the harbor. A simple decimal error ultimately delayed the project by years and nearly doubled the original budget to approximately four billion euros.

The S-80 incident underscores a vital ethical imperative in industrial modeling: the necessity of systemic sanity checks. Models must be designed to be error-sensitive, and the organizational structures surrounding them must guarantee that foundational abstractions are independently validated before physical construction begins.

A parallel catastrophe occurred on June 4, 1996, during the maiden flight of the Ariane 5 rocket. Approximately 37 seconds after liftoff, the rocket abruptly veered off course, broke apart under immense aerodynamic stress, and exploded, destroying four highly valuable research satellites.

The root cause was a data conversion failure within the inertial reference system. A software routine attempted to convert a 64-bit floating-point number representing

horizontal velocity into a 16-bit signed integer. This specific piece of software had been ported almost unchanged from the older Ariane 4. However, the Ariane 5 possessed a significantly different acceleration profile and reached much higher horizontal velocities early in its flight. The velocity value exceeded the maximum capacity of a 16-bit integer ($32,767$), triggering an integer overflow. This caused the primary navigation system to crash, immediately followed by the identical, redundant backup system. The main on-board computer misinterpreted the resulting diagnostic error codes as valid flight data, commanding an extreme, fatal course correction.

The Ariane 5 disaster represents a classic failure of software reuse. In industry, it is standard practice to reuse proven mathematical algorithms and software modules to save costs. However, doing so without rigorously re-evaluating the operational boundaries and physical realities of the new environment constitutes an ethical failure. A mathematical model is only valid within its explicitly defined parameters; abstracting away the physical context of the new rocket transformed a reliable piece of code into a mechanism of destruction.

Systemic Bias in the Machine

While physical engineering failures often end in visible catastrophes, industrial mathematics increasingly governs the digital and financial realms, where errors are invisible and cumulative. In these environments, mathematical models are executed millions of times per second. Here, the ethical focus must shift from single catastrophic failures to the insidious nature of systemic bias, where a mathematically "minor" choice compounds into a macro-economic distortion.

A classic illustration of this phenomenon is the Vancouver Stock Exchange (VSE) index crash of 1982. In January of that year, the VSE established a new stock index, initializing it at a baseline value of $1,000.000$. Over the next 22 months, the market experienced general growth, and the participating companies were performing well. Paradoxically, however, the index steadily plummeted, reaching a low of approximately 524 by late 1983.

The collapse of the index was not an economic reality, but a mathematical artifact. The index was recalculated and updated after every single transaction, amounting to about 3,000 recalculations per day. The programmers who implemented the algorithm made a fateful decision regarding precision: rather than rounding the newly calculated index value to the nearest third decimal place, the software simply truncated it (cutting off the subsequent digits).

In isolation, truncating a number rather than rounding it introduces a microscopic error: a loss of up to $0.00099...$ points per transaction. In pure numerical analysis, the difference between rounding and truncation is often a mere implementation detail. However, in a high-frequency applied setting, truncation ceases to be neutral; it creates a strict, unidirectional downward bias. With thousands of trades happening daily, this systematic bias compounded relentlessly, artificially wiping out nearly half of the index's perceived value and distorting the financial reality of the exchange. When the error was finally identified and the algorithm was corrected to round properly, the index immediately jumped from 524 to its true value of $1,098$.

For the mathematician, the VSE incident highlights that algorithmic precision is an ethical issue. When designing systems that operate at massive scales, developers must recognize that mathematical models can

systematically disadvantage certain outcomes or populations if numerical biases are ignored. The choice of a numerical method is not just a technical parameter; it structurally dictates the fairness and accuracy of the system over time. Modelers bear the ethical responsibility to rigorously analyze the long-term, asymptotic behavior of their algorithms to ensure they do not systematically distort the reality they are meant to reflect.

Mathematics Shaping Policy and Society

While engineering errors lead to physical collapse and algorithmic biases distort markets, mathematical modeling in economics and data science carries a different kind of weight: it directly shapes public policy and human welfare. In these domains, mathematical models are frequently deployed to lend an aura of irrefutable objectivity to highly political decisions, a phenomenon often referred to as "mathwashing."

One of the most consequential examples of this occurred in the wake of the 2008 global financial crisis. In 2010, a highly influential macroeconomic paper, "Growth in a Time of Debt," was published. It asserted a stark mathematical threshold: when a nation's government debt exceeds 90% of its Gross Domestic Product (GDP), economic growth plummets, often turning drastically negative.

This specific numerical threshold was immediately seized upon by politicians and policymakers globally. It served as the primary, "objective" justification for implementing severe austerity measures across Europe and the United States, leading to massive cuts in public spending, infrastructure, and social services. The mathematical model provided the ultimate political shield: austerity was not presented as a debatable ideological choice, but as an inescapable mathematical necessity.

However, three years later, researchers attempting to replicate the study's findings discovered a shocking flaw. Alongside highly questionable weighting methods, the most glaring error was a rudimentary spreadsheet mistake. In the central data file calculating the averages, an aggregation formula merely covered rows 30 through 44, completely omitting rows 45 through 49. This simple coding error accidentally excluded five key countries from the dataset. Once the formula was corrected to include all available data, the dramatic threshold disappeared; countries with over 90% debt actually showed a respectable average growth rate of 2.2%, rather than a recessionary decline.

This case serves as a stark warning. The error itself was technically trivial—a basic tool-handling mistake—but its societal consequences were profound, directly affecting the welfare of millions. The ethical takeaway is clear: when mathematical models dictate public policy or allocate resources, transparency, reproducibility, and open data are not optional academic courtesies; they are strict ethical imperatives. When we fail to open our models to rigorous, independent public scrutiny, we allow simple human errors to masquerade as the immutable laws of nature.

The Observer Effect: Gaming the Mathematical System

Thus far, we have examined mathematical models applied to physical engineering, financial markets, and macroeconomic policy. But what happens when mathematical modeling is turned inward, applied directly to the professional ecosystem of researchers themselves?

The attempt to mathematically model and quantify "academic quality" or "research impact" provides a profound lesson in the ethics of modeling human systems. The introduction of metrics such as the h-index or journal impact factors was originally intended to create an objective, transparent, and purely numerical standard for evaluating careers, allocating funding, and awarding prestige.

However, the widespread adoption of these metrics serves as a perfect illustration of Goodhart's Law, which posits that as soon as a measure becomes a target, it ceases to be a good measure. When we model physical phenomena, the universe does not change its laws to spite the equation. But when we model human performance, the model never acts merely as a passive thermometer; it actively changes the temperature of the room.

Once career survival and institutional funding became strictly tied to citation counts and publication volume, researchers inevitably adapted their behavior to optimize their scores. This algorithmic pressure has led to systemic shifts that often undermine the actual quality of science. A prime example is the rise of *Salamitaktik*: the practice of slicing a single, comprehensive research project into the smallest possible publishable units simply to maximize publication counts. Even more detrimental to scientific integrity is the formation of citation cartels, where networks of researchers or journal editors systematically and artificially cite each other's work to inflate their respective metrics. The mathematical model, intended to objectively measure quality, ends up distorting the very reality it sought to evaluate.

This provides a crucial ethical lesson in feedback loops. When we design models that evaluate human systems—whether for academic performance, employee productivity, or consumer credit scoring—we must anticipate that the human subjects will react to, and potentially game, the algorithm. Ethical modeling in this context requires the humility to understand that we are not simply measuring a system; we are actively engineering the incentive structures that govern human behavior.

Conclusion: A New Standard for Industrial Mathematics

The examples discussed—from submarines and rockets to stock markets, economic policy, and our own academic metrics—demonstrate a fundamental truth: mathematics in industry is never strictly neutral. Every phase of mathematical modeling involves implicit value judgments. An abstraction that ignores physical constraints, systematic biases, or human behavioral feedback loops is not merely mathematically incomplete; it is ethically flawed.

As applied mathematicians, we must abandon the illusion that the language of mathematics automatically confers absolute objectivity upon our models. When an algorithm operates at a massive scale, allocates resources, or dictates public policy, numerical precision ceases to be a purely academic concern and becomes a profound societal responsibility.

As members of the European Consortium for Mathematics in Industry, we operate at the critical intersection of rigorous theory and messy, high-stakes reality. This position demands a new standard of professional practice. We must foster a culture of continuous critical reflection and "error-friendliness," where independent plausibility checks are institutionalized, the limitations of our models are communicated transparently, and algorithms are constantly tested against the physical and social realities they seek to represent.

Ethical modeling is not about achieving mathematical perfection; it is about acknowledging the limits of our abstractions. Ensuring that our models serve reality, rather than forcing reality to fit the model, is not a constraint on industrial mathematics, on the contrary, it is the highest expression of its professional integrity.

Further Reading: The case studies presented in this article—along with a much broader exploration of further topics regarding the moral consequences of mathematical abstractions—are discussed in extensive detail in the authors' forthcoming book: Bátkai, A., & Nickel, G. (2027). Der entscheidende Fehler – Mathematische Modelle, Verantwortung und ihre moralischen Folgen [The Decisive Error – Mathematical Models, Responsibility, and Their Moral Consequences]. Springer Nature, to appear.

András Bátkai

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Geometrical acoustics based aberration correction for layered media in ultrasound imaging

Medical ultrasound imaging is a widely used, fast, inexpensive, and non-invasive diagnostic and therapeutic tool in modern medicine. The standard imaging method relies on the assumption that the underlying medium is homogeneous, with a fixed sound speed of 1540 m/s. In layered tissues, such as those encountered in abdominal imaging through fat and muscle, or transcranial imaging through the skull, this assumption breaks down, causing the reconstructed images to be blurred and geometrically distorted. In collaboration with GE HealthCare and as part of the Christian Doppler Laboratory MaMSi, we have developed a geometrical-acoustics-based focusing algorithm that corrects for such aberrations by tracing acoustic rays through a layered model of the underlying medium. Simulations show that our method's focusing delay errors are below $\lambda/32$, i.e., insignificant for imaging applications. Furthermore, a phantom-based experimental setup demonstrates measurable improvements in lateral resolution compared to currently used methods.

CD Laboratory MaMSi

JKU

ÖAW RICAM

Background on ultrasound image formation

A medical ultrasound transducer is a linear or curved array of typically $M \approx 128$ to 256 piezoelectric elements, each of which can both emit and detect acoustic pressure waves in the megahertz range. An image is acquired by transmitting a sequence of pulses, receiving the back-scattered acoustic waves on every element, and reconstructing the image pixel-wise using these received signals.



The standard reconstruction method, *delay-and-sum beamforming* [1], sums the recorded signals coherently at the point in time at which an echo from the image pixel P_N is expected to arrive. This method is called *focusing the received signals*. For this, the *Time of Flight* (ToF) from every transducer element $P_{0,m}$ to the image pixel P_N is required. Using the standard assumption of a homogeneous medium with constant sound speed $c_0 = 1540$ m/s, this ToF is calculated as

$$\mathrm{ToF}(P_{0,m}, P_N) = \frac{|P_N - P_{0,m}|}{c_0},$$

This time of flight is subsequently used to calculate *focusing delays* (Δt_m), which the *Homogeneous Medium Focusing Algorithm (HMFA)* uses to focus the received signals:

$$\begin{aligned}\Delta t_m = &\max\{\mathrm{ToF}(P_{0,\ell}, P_N) \,|\, \ell = 1, \ldots, M\} \\ &- \mathrm{ToF}(P_{0,m}, P_N)\,.\end{aligned}$$

In the context of real-time medical ultrasound imaging, this expression has to be evaluated on the order of 10^9 times every 20 ms, which makes HMFA the standard choice on current scanner hardware. Additionally, in typical imaging settings, assuming the underlying medium to be homogeneous is a good approximation, since the sound speed variations in human tissue, especially soft tissue, are usually mild. However, sound speeds do vary across tissues: fatty tissue (≈ 1450 m/s), most soft tissue (≈ 1540 m/s), muscle (≈ 1600 m/s), connective tissue (up to 1700 m/s), and bone (on the order of 2500 to 3500 m/s, depending on type). Moreover, even the transducer cover, a plastic cap that shields the array and conducts the ultrasound waves into the body, affects propagation. Depending on the material used, its sound speed ranges from 1000 to 2500 m/s, significantly deviating from soft tissue.

This results in HMFA calculating an incorrect ToF, which leads to incorrect focusing delays. As a consequence, the signals received at different transducer elements are added incoherently, causing degraded image quality, visible as the blurred point targets in the leftmost panel of Figure 1 compared to the ground truth in the rightmost panel. The remaining panels show two refined methods discussed in detail in the following section: *Straight Ray Tracing* (SRT), which accounts for the layered sound speeds along straight rays, and our *Geometrical Acoustics Focusing Algorithm* (GEOFA), which additionally accounts for refraction at medium boundaries [2].

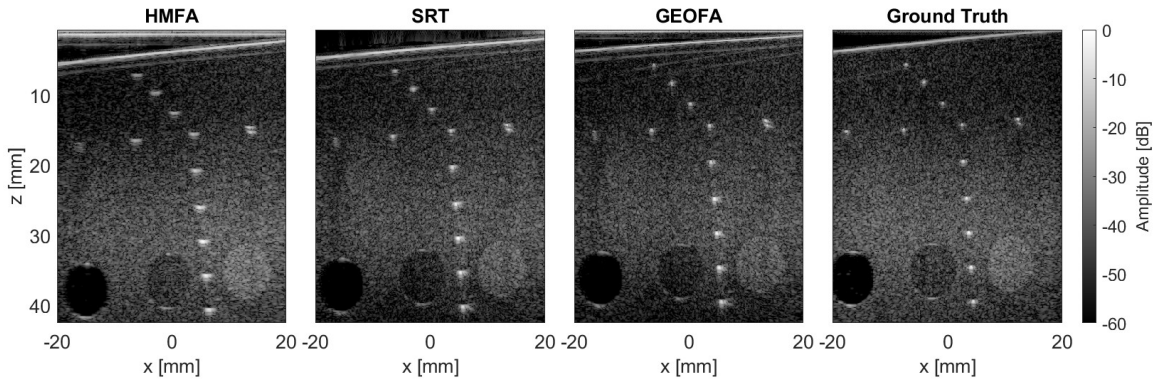


Fig. 1: Aberration effects caused by a wedge-shaped aberrator on images reconstructed with three different ToF calculations: HMFA (Left, standard constant c_0 method), SRT (Center-Left, layered sound speeds along straight rays) and GEOFA (Center-Right, additionally accounting for refraction). For comparison, a ground-truth image (Right) is obtained by removing the aberrator.

Our Geometrical Acoustics Focusing Algorithm (GEOFA)

Our approach refines the ToF calculation [2], so that the calculated ToF becomes a better approximation of the true underlying physical ToF of the acoustic waves through the non-homogeneous medium. Consequently, the resulting focusing delays correct for the aberrations in the reconstructed image. For this, we need information about the underlying medium. Therefore, we assume that the medium consists of known, piecewise-homogeneous layers with known sound speeds and at least once continuously differentiable medium boundaries. For the transducer cover, this assumption is satisfied, since its geometry and acoustic properties are known a-priori. For patient-specific tissue, the layer geometry and the layer-wise sound speeds have to be reconstructed before our method can be applied.

Under these assumptions, the ToF from a transducer element to an image pixel can be calculated using geometrical acoustics as follows: In a homogeneous medium layer, the geometrical acoustics path, also called acoustic ray, is a straight line (center panel of Figure 2). At each medium boundary, refraction occurs (right panel of Figure 2) according to Snell's law:

$$c_{n+1} \sin \theta_n \;=\; c_n \sin \theta'_n, \tag{3.1}$$

where θ_n and θ'_n denote the pre- and post-refraction angles, and c_n, c_{n+1} are the sound speeds of the two adjacent layers. An acoustic ray that starts at a transducer element $P_{0,m}$ and ends at the image pixel P_N, see the center panel in Figure 2, is therefore uniquely determined by its sequence of intersections $(P_{n,m})_{n=1}^{N-1}$ with the medium boundaries. The associated ToF is the sum of the layer-wise ToFs, i.e.,

$$\mathrm{ToF}(P_{0,m}, P_N) \;= \sum_{n=1}^{N} ToF(P_{n-1,m}, P_{n,m}) \tag{3.2}$$

$$= \sum_{n=1}^{N} \frac{|P_{n,m} - P_{n-1,m}|}{c_n} \,. \tag{3.3}$$

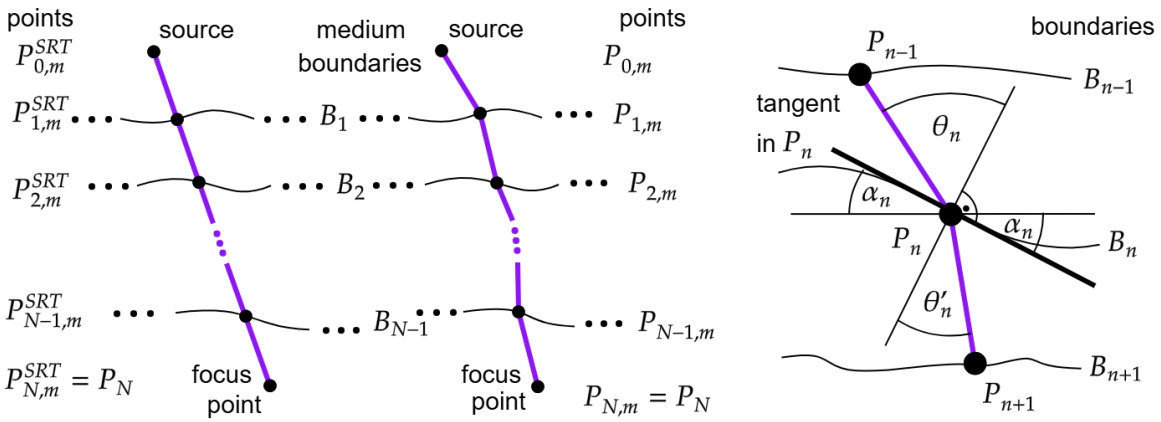


Fig. 2: Geometry of the SRT and GEOFA. Left: SRT connects each transducer element image pixel pair with a straight line, which is intersected with the medium boundaries B_n to obtain points $P_{n,m}^{\mathrm{SRT}}$. Center: GEOFA replaces the straight-line geometry with the physically correct refracted ray, intersecting the boundaries at $P_{n,m}$. Right: Local geometry at a boundary B_n, with pre- and post-refraction angles θ_n, θ_n' measured against the boundary normal, and local boundary slope α_n.

A previously introduced, widely used refinement of HMFA, *Straight Ray Tracing* (SRT, left panel of Figure 2) [3, 4, 5], evaluates (3.3) along the straight line connecting $P_{0,m}$ and P_N, that is, SRT accounts for the layered sound speeds but ignores the refraction at medium boundaries. In comparison, GEOFA retains the physically correct refracted ray geometry. Both GEOFA and SRT need to determine their respective medium boundary points $(P_{n,m})$. For SRT, this requires the intersection of a line with each medium boundary. For GEOFA, however, the medium boundary points are implicitly defined by a coupled system of $5 \cdot (N-1)$ nonlinear equations for the variables θ_n and θ_n', the local boundary slope α_n, and the two boundary point coordinates [2].

In the absence of total reflection, we have shown the existence of a solution to GEOFA's underlying system of equations. Furthermore, the solution is unique provided that the medium boundaries are not too steep [2]. Having established existence and uniqueness of solutions to the GEOFA system, we now turn to its quantitative validation.

Numerical and Experimental Results

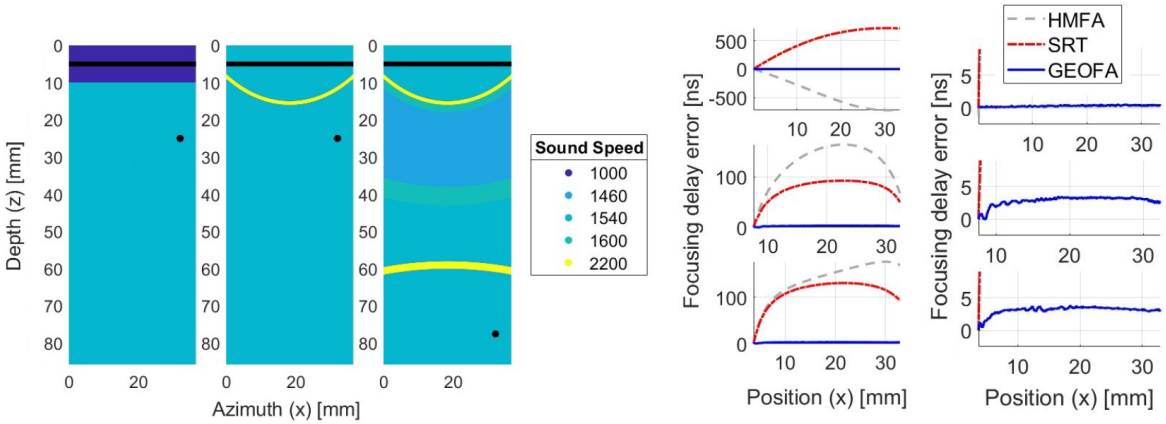
The numerical validation used the acoustic toolbox k-Wave [6]. Acoustic wave propagation through multiple layered media, which are shown on the left of Figure 3, was simulated by emitting a spherical wave from each point source (black dots in the figure), and recording the resulting pressure on a linear array of receivers (black line on top of the simulation area). From these recordings, the true ToF and the corresponding focusing delays were extracted and compared with values obtained via HMFA, SRT, and GEOFA.

Across all media, GEOFA was the most accurate of the three methods. In configurations with straight medium boundaries, GEOFA's ToF errors were below one nanosecond. In configurations with curved medium boundaries, they remained below 5 ns. These higher error levels for non-straight medium boundaries are attributed to discretization effects in the k-Wave simulation and to wave-like phenomena that cannot be captured by the geometrical acoustics model. SRT, in contrast, produced ToF errors up to two orders of magnitude larger, since it ignores

refraction at medium boundaries. HMFA's errors were larger still, since the constant sound-speed assumption itself is violated [2].

The corresponding focusing-delay errors, shown on the right of Figure 3, followed the same pattern. GEOFA achieved focusing-delay errors below 6.25 ns across all settings, which corresponds to $\lambda/32$ at a 5 MHz center frequency and is thus well below the level relevant for image quality. SRT and HMFA again exceeded this threshold by up to two orders of magnitude, since their ToF errors directly caused increased focusing delay errors.

The experimental validation used a wedge-shaped aberrator placed between a linear-array transducer and a tissue-mimicking phantom containing wire targets [2]. Channel data were exported and reconstructed with HMFA, SRT, and GEOFA (first three panels of Figure 1). A ground-truth image, shown in the rightmost panel of Figure 1, was obtained by removing the aberrator. SRT already produced a noticeable de-blurring relative to HMFA, and GEOFA further improved image quality. For a quantitative evaluation, lateral profiles were extracted at all wire targets. A representative selection is shown in Figure 4. Measured at -5 dB, these profiles show GEOFA significantly outperforming SRT for targets shallower than 15 mm and performing comparably elsewhere, closely matching the ground truth throughout [2]. At -15 dB, a gap between GEOFA and SRT persisted across all depths, and both methods deviated from the ground truth for targets deeper than 15 mm, although both remained substantially closer to it than HMFA. The residual deviation visible in target 5 and the enlarged lateral profile widths at -15 dB are the subject of ongoing investigation.



Left: Three layered media used for the k-Wave simulations, with point sources marked as black dots and the receiver array as a black line on top of each simulation domain. Right: Corresponding focusing-delay errors of HMFA, SRT, and GEOFA. The three rows correspond, from top to bottom, to the three media on the left. In each row, the left subplot shows all three methods, while the right subplot rescales the y-axis to make GEOFA's residual errors visible.

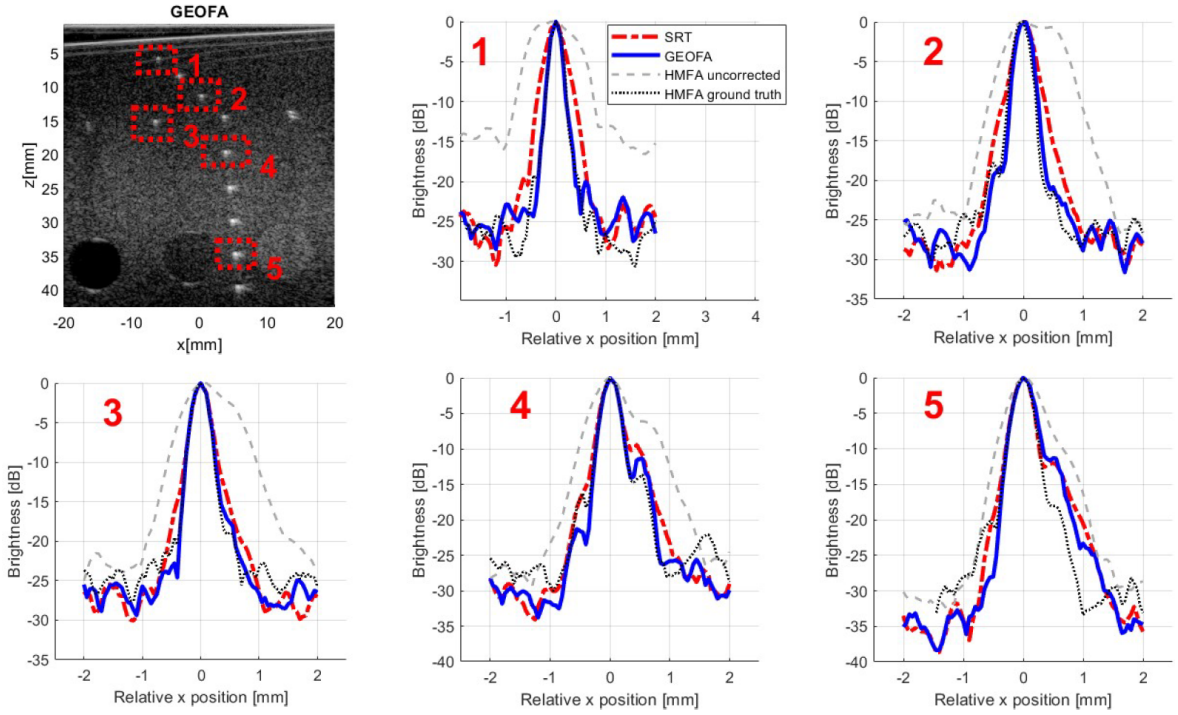


Fig. 4: Beam profiles evaluated at multiple wire targets.

Outlook

In the previous section, we outlined in detail the accuracy of our method. A caveat concerns computational cost. In real-time medical ultrasound imaging, the focusing delay calculation has to be performed on the order of 10^9 times per frame, i.e., every 20 ms, to achieve a 50 Hz frame rate. HMFA meets this requirement, since it involves only a simple closed-form calculation. GEOFA, in contrast, requires the solution of an implicit system of $5 \cdot (N-1)$ nonlinear equations per element-pixel pair, which significantly exceeds the currently available computational capacity. In view of the substantial improvements in focusing accuracy and aberration correction reported above, we are currently developing an approximate refraction correction method that retains most of GEOFA's benefits at a computational cost compatible with real-time medical ultrasound imaging.

A second open direction concerns the assumption that the underlying medium is known. While this is the case for the transducer cover, the medium geometry and layer-wise sound speeds of patient-specific tissue have to be reconstructed before GEOFA can be applied. Combining medium reconstruction with refraction-corrected focusing into a single algorithm is therefore a key step toward general clinical applicability and is the subject of ongoing work within the CD Laboratory MaMSi.

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Optimal strategy for trail running with nutrition and fatigue factors

Trail running has become increasingly popular in recent years, and the number of runners participating in off-road races continues to grow. Unlike road running, trail races often take place in the mountains or on uneven terrain, where athletes face steep climbs and descents, technical surfaces, and changing weather conditions. These challenges mean that runners must adapt their pace and energy use throughout the race.

History

In 1973, Joseph Keller developed a simple model that describes the motion of the runner (Newton's second law) and changes in energy level (difference between the aerobic energy production rate of the runner and their power output) [1]. The objective was to maximize the distance covered in a given time, with natural constraints on energy levels and the maximal propulsive force that the runner can exert. The optimal strategy turned out to be an initial acceleration, using the maximal force, followed by a period of keeping an almost constant pace and deceleration in the final part, when the energy level drops to zero. With this simple framework, Keller was able to reproduce the world records on distances from 100 meters to 10 kilometers with good accuracy. Throughout the years, many authors have extended this model by adding a variety of factors such as fatigue [2], the impact of the wind [3], the restoration of energy while slowing down [4] or competition with a second runner [5]. However, all of these works were focused on short- to medium-distance on a flat route.

Model

Strategy optimization for a race taking place in the mountains, especially on a long distance, was still terra incognita for mathematicians. In our study [6] we provided a generalization of the Keller's model, adapting it to better describe the specific demands of trail running. The problem is stated as an optimal control problem. The objective is to minimize the race time T on a track of length D with correct control of the propulsive force per unit mass $f = f(t)$. These two quantities are

related by:

$$D = x(T) = \int_0^T v(t)dt,$$

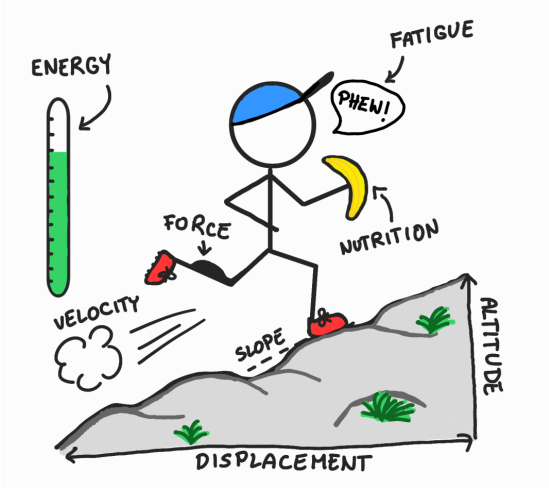
where $v = v(t)$ is the velocity of the runner and $x(t)$ is the distance covered by the runner in time t. Similarly to Keller's work, the central equations of our model are those that describe the motion and energy of the runner. Diagrammatically, they can be stated as follows:

$$\begin{aligned} \text{net force} \quad = \quad & \text{propulsion} - \text{gravity} \\ & - \text{internal dissipative force} \\ & - \text{drag} \end{aligned}$$

$$\begin{aligned} \text{energy change} \quad = \quad & \text{supply} - \text{work} \\ & + \text{nutrition} - \text{fatigue} \end{aligned}$$

The new factors included in the model are:

- impact of gravity - making it easier to run downhill than climb uphill.
- impact of altitude above sea level: in high altitudes, air is low in density, and it is harder to provide oxygen to cells. We limit the energy available to the runner based on altitude.
- fatigue - prolonged exercise leads to increased fatigue. We assume that the fatigue increase is proportional to the work rate performed by the runner, corrected by the factor of GAP, which accounts for the varying difficulty of running at different slopes. GAP (Grade Adjusted Pace) is a concept introduced by Strava that enables comparing the effort required to run at given pace, at given slope, with its equivalent on the flat route.
- nutrition - nutrition is an essential factor in long-distance races. Carbohydrate ingestion during prolonged moderate to high intensity endurance exercise has been shown to increase capacity and performance.



Model validation

To check the model validity, we decided to compare the race accomplishment times returned by our model with the route records. The physiological parameters, such as VO2max, body mass, initial energy level, and maximal force that the runner can use, were calibrated to mimic a typical elite male runner. The races ranged from verticals to half-marathons and marathons. Therefore, we are able to test our model in a variety of running scenarios. The results are presented in Table 1.

When discussing the results of the model, we need to keep in mind that compared to flat-road competitions, such as athletic events or road races, it is much harder to estimate the finish time in trail races. In natural terrain, obstacles can occur, such as fallen trees, mud, or slippery rocks. Also, the weather plays a very significant role - rain or snow might cause severe or even extreme conditions, especially on the steep parts, while running both uphill and downhill. In addition, the type of surface has a big impact on the results; it is much easier to run on a gravel road than on a pavement consisting of large and irregular rocks. It is then hard to compare two route segments, even with the same slope, as depending on the surface type, the amount of time needed to accomplish them might be

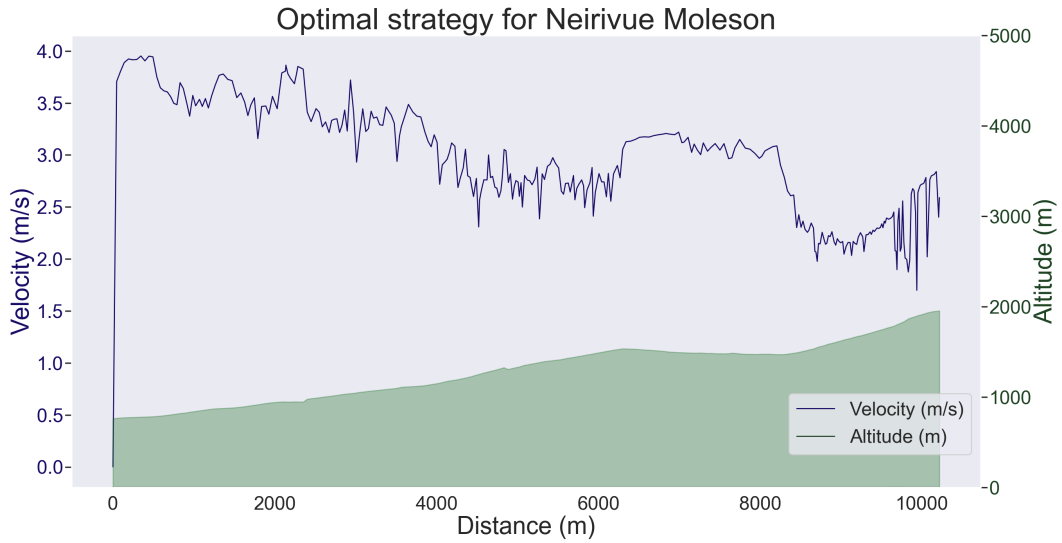
significantly different. Despite these difficulties, the agreement between the finish times returned by the model and the route records is very good. The estimated finish times are very reasonable in all the cases and might serve as a good guide for the runner. Predominantly, the difference between the estimated finish time and the route record does not exceed 5 minutes and the relative error does not exceed 5%, which is an impressive result.

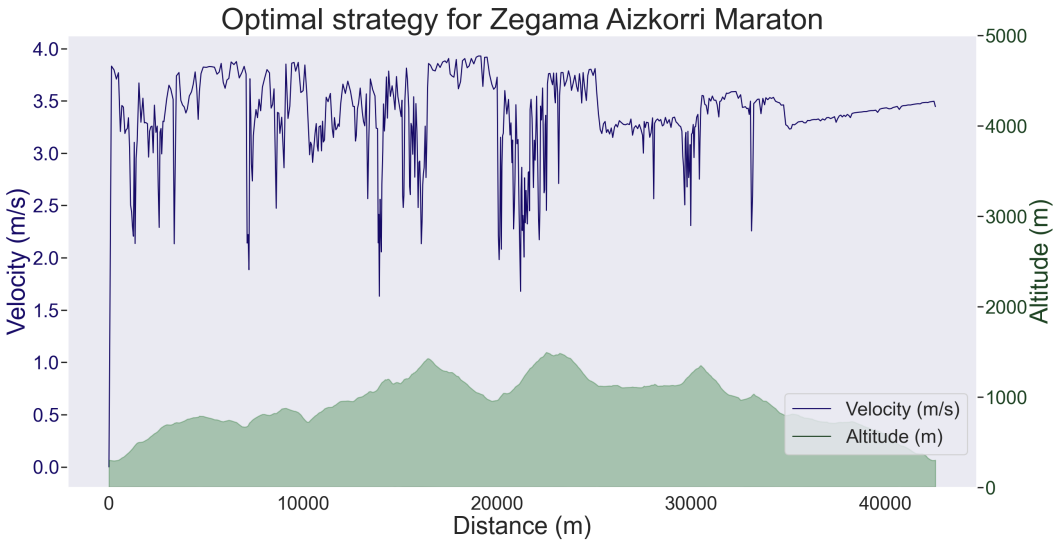
Table 1: Comparison of route records and calculated finish times.

Race	Route record	Numerical result	Relative error
Il Golfo dell'Isola	1 h 56 min 15 s	1 h 56 min 37 s	0.32%
Quad Bomber	1 h 53 min 47 s	1 h 58 min 50 s	4.44%
Mammoth 26k	1 h 54 min 48 s	2 h 06 min 04 s	9.81%
Olla de Núria	2 h 04 min 16 s	2 h 03 min 28 s	0.64%
Stranda Fjord Trail Race	2 h 24 min 02 s	2 h 21 min 13 s	1.96%
La Skyrhune	1 h 52 min	1 h 41 min 02 s	9.79%
Chiemgau Trail Run	3 h 47 min 34 s	3 h 46 min 10 s	0.62%
Zegama Aizkorri Maratón*	3 h 36 min 40 s	3 h 34 min 27 s	1.02%
Marathon du Mont Blanc	3 h 35 min 04 s	3 h 36 min 31 s	0.67%
WMTRC Classic Uphill	46 min 51 s	46 min 22 s	1.03%
Neirivue Moléson	55 min 41 s	58 min 28 s	5.00%

Optimal strategies

Although finish times give us a good idea about model validity, the most important outcome is velocity strategy. It can serve as a guidance for the runner, helping in deciding when to put more effort and when to relax to restore energy. Below we present the strategies for two different races: vertical (Neirivue Moleson race) and marathon (Zegama Aizkorri Maraton).





The strategies strongly depend on the slope. In the case of Neirivue Moléson, the velocity gradually decreases over the first 6 kilometers, reflecting the sustained climb. This is followed by a noticeably faster pace in the downhill section. The race then ends with a final "all-out" acceleration. Zegama Aizkorri Maraton, which is a much longer race, with multiple uphill and downhill sections, has a much more complex pacing strategy. What is particularly interesting, however, is that in the final kilometers the velocity increases, showing a similar end-of-race behaviour to that observed in Neirivue Moléson.

Conclusion

Our generalization of the classical Keller model can be successfully applied to provide an optimal running strategy even for long races on a varying and difficult terrain. We have shown that, when applied to real-world data of several routes of various categories, our model provides an accurate estimate of the finishing time. Taking into account the difficulty of the terrain as well as unpredictable weather conditions, we can conclude that the model performs very well for many different scenarios. Since our computations are generic, the results obtained might be improved, for example, by taking into account the following points.

- Providing detailed information about the runner, such as the exact value of their VO2max and body mass instead of some generic values.
- Including the type of surface in the calculations. Although such data are hard to obtain, taking into account the impact of the pavement will make the model much more realistic.

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4

Featured People

More Than a Research Project: How the European PhD Network “THREAD” Connects People

THREAD - Joint Training on Numerical Modeling of Highly Flexible Structures for Industrial Applications, was an innovative European Training Network (ETN) funded by the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement. Running from October 2019 to March 2024, the project brought together a unique consortium of universities, research organizations, and 12 industrial partner organizations. During the development of the project, THREAD supported 14 Early Stage Researchers, providing them with comprehensive training in state-of-the-art mathematical formulations and modeling techniques. The researchers benefited from close collaboration with industry partners through a dedicated program of research secondments, bridging the gap between theoretical academic science and practical applications. Here we get to know a bit closer 4 of their supported researchers.

Denise Tumiotto – Understanding the World Through Equations and Knowledge as a Lifelong Asset

For Denise Tumiotto, mathematics is a key to understanding the world: “In my view, only questions involving morality and emotions resist mathematical modeling. Matter itself – understood in the physical sense – can always be described through mathematics. Each motion, each force, each physical element can be described via an equation”, she says.



Denise studied “Mathematical Engineering” in Turin, bridging theory and application, and was drawn to THREAD for its interdisciplinary setup: “I liked the possibility to have a mixed environment of engineers and mathematicians.”

A key experience was her international research stay within the network, which broadened both her scientific and personal perspective. Today, she works at fleXstructures Italy, a Fraunhofer ITWM spin-off, applying the IPS Cable Simulation software to real industrial problems. She had already completed an internship there during her PhD and values the close link between research and application.

For her, knowledge is central: “I value knowledge the most in a person and in society. It means that if I do not know something I feel like something is missing and I need to discover it and understanding it.” Her guiding principle: “Scientific research is the only possible way to improve ourselves.” Her advice to future doctoral students is also consistent with this attitude: “At the beginning it may feel like you do not know enough”, she says. “But keep studying and your knowledge will grow together with you.”

Davide Manfredo – “Don’t Waste Time and Energy Comparing Yourself to Others”

For Davide Manfredo, research became a clear path only late during his studies. After mathematics studies in Turin and a shift toward applied mathematics, including international stays in France, Thailand, and Spain, he joined THREAD at Fraunhofer ITWM to work on modeling highly flexible structures. He highlights the value of exchange in the network: “I realized how many people work on the same topic but from different perspectives.”



Starting his PhD during the pandemic made collaboration initially difficult, but over time

he built strong connections he still values today.

Today he works at fleXstructures GmbH developing simulation software for cables. “Pure research would not have worked for me,” he emphasizes. There, he develops and maintains software for the simulation of cables – exactly the field that already occupied him during his doctorate. He now uses this knowledge in practice every day. “I also learned German during my PhD, and I now work totally in German.”

What Manfredo still finds particularly fascinating today is that real phenomena can be described so precisely with mathematical models that they can be simulated on a computer. “Every time I think about it seems unbelievable,” the mathematician says enthusiastically. It is precisely this mix that appeals to him. “I like the fact that my job varies almost every week,” says Manfredo.

His advice for future doctoral candidates: “Do not waste time and energy comparing your results and accomplishments with others. Every path is unique, as well as every student and supervisors, research group and research topic. Focus on getting something done everyday and trust that results will come sooner than expected. Do not forget to enjoy your free time, it is important to recharge your batteries.”

Ergys Çokaj – Research as a Community: Where the Textbook Ends

For Ergys Çokaj, the decision to pursue research emerged gradually during his mathematics studies at the University of Camerino in Italy. He became increasingly drawn to problems without ready-made solutions: “What fascinated me most were the problems where the textbook stopped and someone actually had to do the extra work to find an answer”, he recalls.



This curiosity led him to the European doctoral network THREAD. As a Marie Skłodowska-Curie Early Stage Researcher at NTNU in Trondheim, he worked on structure-preserving numerical methods and Machine Learning approaches for mechanical systems – in essence, developing simulation methods that respect physical laws while integrating data-driven techniques.

What attracted him most was the interdisciplinary setting: “THREAD was the place where mathematicians and engineers learned to speak each other’s language in order to model the things that bend, twist, and refuse to behave.”

During his PhD, he spent research stays at Friedrich-Alexander-Universität Erlangen-Nürnberg and Martin Luther University Halle-Wittenberg, as well as an industrial placement at TechnipFMC in Norway, working on sensor data and flexible marine risers. These experiences reshaped his view of research: “The defining experience of THREAD was discovering that I belonged to a scientific community. Good research is something you do with people, across countries, languages, and disciplines, not alone.”

Today, he is a Lecturer at Western Balkans University (WBU) in Tirana, teaching numerical analysis, optimization, data science, and Machine Learning, while remaining active in applied research projects such as Sensatolla and Tech2Link. Although his topics have diversified, he sees a consistent thread: “Mathematics allows you to move between very different application areas. The tools change, but the way of thinking remains remarkably similar.”

After completing his PhD in 2024, he chose to return to Albania: “Staying abroad would have been the easier path. Coming back felt like the more meaningful one.” At WBU, he combines teaching, research, and institutional development, aiming to foster a culture of curiosity and collaboration. Looking back, he emphasizes the non-linearity of research progress: “There will be long periods when nothing seems to work. That is not a sign that you are failing. It is simply what research feels like from the inside.” His advice is simple and steady: “Good work is honest work, even if small. Small steps beat standing still.”

Andrea Leone – Between Mathematics, Data, and Engineering Practice

Andrea Leone combined physics and mathematics early on: He studied physics (Bachelor’s) and mathematical engineering (Master’s) at the University of L’Aquila, consistently working at the intersection of theory and application. During his studies, he delved into optimization, numerical methods, and mathematical modeling, developing a growing interest in data science and machine learning, which he also explored in his master’s thesis. An Erasmus exchange at NTNU in Trondheim further broadened his perspective – including through work on neural networks from the perspective of optimal control problems.



In August 2020, he joined the European doctoral network THREAD, which offers him exactly the right combination: structure-preserving numerical methods, geometric numerics, and data-driven approaches for flexible structures such as beams and cables.

The international and interdisciplinary collaboration within the network is particularly formative for him: mathematicians and engineers work together on fundamental modeling questions and gain exposure to different methodological perspectives. THREAD thus becomes a place for him to delve deeper into scientific questions within a European context.

Today, his profile combines mathematical modeling, numerical analysis, and Machine Learning. This combination shapes his work on complex technical systems where mathematical precision and practical feasibility must come together.

The international research environment had a lasting impact on Leone’s way of working and strengthened his flexibility. For him, research means above all dealing with uncertainty and remaining open to different approaches – a skill he considers central to his professional life today.

THREAD also had a direct influence on his career path. During his PhD, he completed an industrial secondment at TechnipFMC, a global engineering and technology company for the energy sector. The experience gave him valuable insight into how mathematical models and numerical simulations are applied to real-world engineering challenges.

After completing his doctorate at NTNU, he joined TechnipFMC, where he now works on the simulation of subsea systems and the definition of safe operational windows for offshore installation activities. Looking back, Leone sees a clear connection between his academic work and his current role: the combination of mathematical modelling, numerical methods, and data-driven approaches continues to shape his work on complex engineering systems where scientific rigor and practical applicability must go hand in hand.

Interviewed by Esther Packullat

Fraunhofer Institute for Industrial Mathematics ITWM, Kaiserslautern, Germany

5

Projects and Case Studies

Human-Centered Mathematics at Fraunhofer ITWM

Mathematical models are used in a wide range of human-centered fields, including healthcare, product design, and the ergonomic design of work processes. At Fraunhofer ITWM, researchers combine methods from various disciplines to address these areas: Machine Learning (ML), statistics, optimization, and multibody simulation. This article presents two different application examples: mathematical models for diagnostic support of rare diseases and digital human models for predicting active, deliberate motions to improve vehicle design and safety. In addition, Fraunhofer ITWM is developing concepts for studying the behavior of pedestrians and vehicle occupants using a combined test setup consisting of a driving simulator and a VR system [1], as well as methods for improving medical care, such as the optimization of patient-specific treatment plans and the simulation-based design of medical devices [2].

Detecting Rare Diseases Faster Thanks to Mathematical Models

Rare diseases affect more people than one might think, but they are often difficult to diagnose [3, 4]. On average, it takes years for patients to receive a diagnosis. In this project, our researchers are working with the pharmaceutical company Chiesi to develop a mathematical support tool that will help doctors identify rare diseases more quickly and test for them in a targeted manner. There are over 8,000 known rare diseases worldwide – no doctor will be familiar with all of them. This is where our research comes in: the tool we are developing together with Chiesi calculates the probability of various rare diseases based on the symptoms entered. In this way, we support medical professionals in narrowing down possible clinical pictures and initiating targeted tests.

What Defines Rare Diseases?

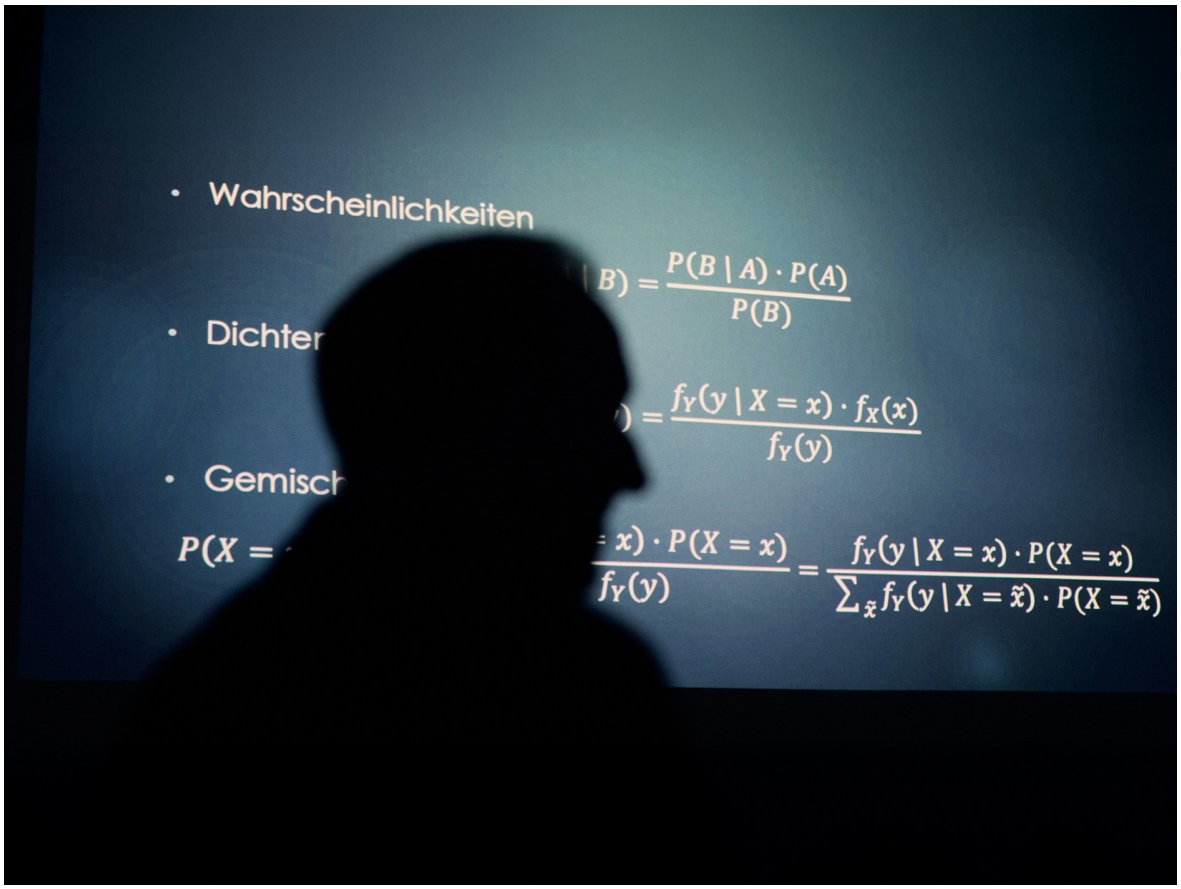
Rare diseases affect more people than one might think – but that is only one of their characteristics. Many have genetic causes and often manifest early in life. They frequently impose severe limitations, shorten life expectancy, and remain incurable to this day. Furthermore, effective therapies are lacking for many of these diseases. The goal of the project is to ensure

that those affected receive the correct diagnosis and thus the appropriate treatment more quickly.

Our Methodological Approach: Bayesian Statistics and Monte Carlo Simulation

For the diagnostic tool, we combine Bayesian models with Monte Carlo algorithms. These mathematical methods make it possible to calculate reliable probabilities even with incomplete or uncertain information. This allows patient data, known symptom distributions, and prior medical knowledge to be intelligently linked. The more information is available, the more accurate the results are – meaning that the model can be continuously improved with new medical data.

A particular challenge lies in recording and modeling the symptoms: in the first phase of the project, the team focused on a smaller group of rare metabolic diseases to make the task manageable. For her bachelor's thesis, student Chiara Freitag delved deeply into the symptoms and structured the available data from the perspective of those affected – that is, how symptoms are typically perceived and reported. This is because many patients do not have all the symptoms or are unaware of some of them because they can only be detected through special examinations. The stochastic model must therefore deal with incomplete and uncertain data, while at the same time integrating prior knowledge about symptom probabilities in order to make the most reliable statements possible about probable clinical pictures.



Cooperation Between Universities, Industry, and Research

The project was developed in close collaboration with Bielefeld University of Applied Sciences and Arts (HSBI). The bachelor's thesis [5] by Chiara Freitag, who is now a research assistant at Fraunhofer ITWM, laid the methodological foundation. The thesis was supervised by Jörg Horst from HSBI and Jan Hauth from Fraunhofer ITWM.

Based on this research, the pharmaceutical company Chiesi commissioned us to develop the diagnostic tool – the second phase of the project has already been completed.

Benefits and Prospects: Effective Diagnostic Aid

Our approach does not replace a medical diagnosis, but it can significantly shorten the path to one. Doctors receive mathematically sound support in assessing rare clinical pictures, and patients benefit from earlier treatment and thus a better quality of life.

In the long term, the model can be expanded to include additional disease groups and adapted to new data sources. This is a good example of how applied mathematics at Fraunhofer ITWM enables concrete improvements in medical care.

You can find more information about the project here:
www.itwm.fraunhofer.de/rare-diseases

How Mathematical Models Enable More Realistic Accident Simulations

In a traffic accident, it is not only the impact itself that determines possible injuries. The crash is often preceded by a highly dynamic situation, known as the Pre-Crash Phase: the vehicle brakes abruptly, occupants try to brace themselves or react reflexively to the acceleration. These movements change the posture and speed of the limbs and thus significantly influence the risk of injury at the moment of impact [6].

This raises a key question for accident research: How can these movements be simulated realistically?

Two Modeling Approaches – Two Strengths

Today, Finite Element Human Body Models (FE-HBM) are primarily used in numerical crash simulations. These high-resolution models describe the human body with hundreds of thousands of elements and enable detailed biomechanical analysis of stresses, tissue deformations, and possible injuries. Since they are extremely computationally intensive, the duration of the simulation and the simulation of active movements are limited. Multibody System Digital Human Models (MBS-DHM) offer an alternative. They describe the human body as a system of rigid segments connected to each other via joints. Movements can be calculated very efficiently, allowing dynamic scenarios – such as braking maneuvers – to be simulated accurately over a period of seconds. Since the body segments are assumed to be rigid, biomechanical effects such as tissue deformation are neglected, limiting the possibility of statements about injury risks. From a mathematical perspective, both modeling approaches complement each other perfectly:

- Multibody Models are ideal for describing movements.
- Finite Element Models enable detailed analysis of stresses and injuries.

The challenge is to connect both worlds.

EMMA as a Bridge Between Movement and Crash Simulation

The EMMA (Ergo-dynamic Moving Manikin) simulation environment [7] (Figure 1) was developed at Fraunhofer ITWM (in close cooperation with FCC, Göteborg) for this purpose. EMMA is based on a multibody model of the human body and allows efficient calculation of movements under various boundary conditions, such as vehicle acceleration or braking maneuvers.

In the EMMA4Drive project, EMMA was expanded to include an interface for finite element simulations by transferring a FE-HBM to a MBS-DHM [8]. This allows the postures and movements calculated in EMMA to be transferred to the FE-HBM and then reused in a crash simulation. A practical advantage of this approach is that it greatly simplifies the otherwise very time-consuming positioning of the FE-HBM, since the position can be derived directly from a movement predicted by EMMA.

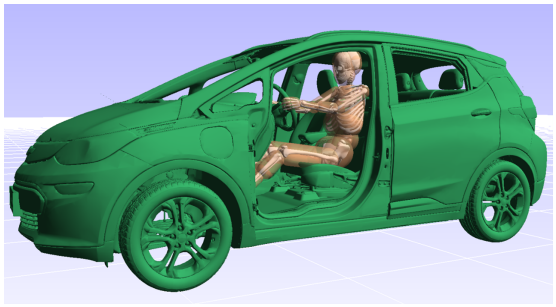


Fig. 1: Simulation environment EMMA: Predicting occupant behavior involving deliberate motions. Vehicle model: Courtesy of fleXstructures and ·D vehicle data courtesy of Caresoft Global.

Project EMMA2HBM: Transferring Movements Between Two Model Worlds

Within the EMMA2HBM project, we investigate how movements from an MBS-DHM can be transferred to a high-resolution FE-HBM. The focus is on comparing two digital representations of the same body: the THUMS® FE-HBM of a 50th percentile male [9] and the MBS-DHM of the EMMA simulation software, which was developed based on the same reference model.

The differences in the modeling approaches described above mean that even with identical body postures, deviations can occur – for example, in the position of individual limbs or in the centers of mass of body regions. The first step of the project is therefore to systematically compare selected body poses and quantify possible differences. Based on this, it is investigated whether movements from the MBS-DHM can be transferred to the FE-HBM. To do this, the FE-HBM is first stabilized under gravity and then excited via time-dependent force vectors calculated by EMMA.

If this approach proves successful, FE-HBM could in future not only be positioned statically but also controlled by calculated movement sequences. This would allow the kinematic efficiency of MBS-DHM to be combined with the biomechanical detail of FE-HBM – a promising step towards more realistic simulations of human movement and stress. The EMMA2HBM project is a study conducted by the Fraunhofer network *Numerical Simulation of Products and Processes*, which we are carrying out in collaboration with Fraunhofer EMI. The network works across institutes on tasks related to the development and improvement of simulation methods.

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Drug Delivery Model for the Assessment of Microcapsules Release Through a Textile Media

The application of microcapsules containing active principles is increasingly being extended, particularly in the field of textiles. In some cases, the use of cosmetotextiles or biofunctional fabrics requires calculating the specific conditions and properties needed to guarantee a minimum dose of the active ingredient. Previous mathematical studies of this phenomenon have typically analysed the individual components of the system in isolation. Our aim is to establish a mathematical approach that treats the three components together — microcapsules, textile and skin — which is much closer to the reality of ensuring effective dosing to the skin. We study the release of the active ingredient from the microcapsules into the textile and its diffusion towards the neighbouring regions of tissue (both effects lumped into a single diffusion coefficient), and from there, through a secondary concentration gradient, its diffusion to the skin–tissue interface. This model is then adapted to the relevant requirements to arrive at a final, tractable mathematical expression

Introduction

Nowadays, competitiveness in the textile sector depends on the ability to understand and meet customers' needs with the incorporation of technological evolution. One of the advances that has been incorporated in this field has been microencapsulation, which has responded to the needs of customers and the opportunity to create new differentiated products with a high added value.

Microencapsulation is based on a method of incorporation and protection of various non-toxic, biocompatible and biodegradable

substances (they may be solid or liquid material) coated by a continuous film of a polymeric material, where its main function is to protect the nucleus from external agents. The intention of microencapsulation is not only that of the protection of core substances, it is also the separation of reagents, controlled release, reduced toxicity and reduced product volatility to be encapsulated.

Understanding the mechanism by which the capsules work and the controlled release of the active ingredient occurs is based on drug delivery principles. Drug delivery works from diffusion phenomena in the polymer-microcapsule system and in the skin. Therefore, each release system requires prior knowledge of the mechanism through the each material. This study aims to model through physically-based equations the drug-delivery phenomena of the active compound from functionalized textiles to the skin.

Governing Equations

The experimental setup considered consists of a thin film of textile where the microcapsules have been previously injected. This film is immobilized at the bottom of an Erlenmeyer flask, which is later filled with a bath of serum that imitates human sweat. If we assume that the textile gets initially soaked in water at a very fast time-scale, we can consider that the active principle starts being released from the microcapsules when the bath is introduced to the flask.

In order to simplify the model, an average concentration of active principle inside the textile has been taken. Thus, an effective diffusion coefficient has been considered, which gathers the combined effect of the diffusion through the microcapsule wall and through the porous textile media. We shall write then the mass conservation in the textile as:

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2},$$

where $c(x,t)$ is the concentration of active principle in the interstitial fluid of the matrix lattice (kg/m^3), and D is the effective diffusion of the active principle in the textile (m^2/s). The diffusion equation also applies in the bath,

$$\frac{\partial c_b}{\partial t} = D_b\frac{\partial^2 c_b}{\partial x^2},$$

where $c_b(x,t)$ is the concentration of active principle in the bath (kg/m^3), and D_b is the diffusion coefficient of the active principle in the bath (m^2/s).

As mapped in Figure 1, the governing equations are subject to boundary conditions across the domains, matching individual configurations from the left-hand substrate base to the upper fluid free-surface.

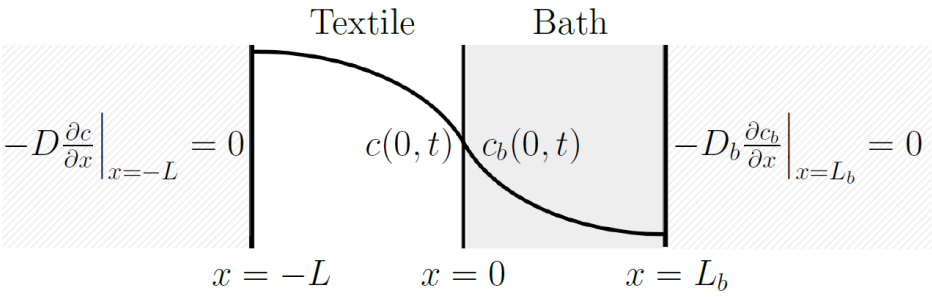


Fig. 1: Spatial configuration schematic illustrating the continuous model domains along with their respective boundary matching laws defined at the outer edges and the interface layer ($x=0$).

If the length of the textile film is L and the length of the bath until the surface is L_b, the left and right boundaries at $x = -L$ and $x = L_b$, respectively, must satisfy a no-flux condition. Namely,

$$-D \left.\frac{\partial c}{\partial x}\right|_{x=-L} = 0, \quad -D_b \left.\frac{\partial c_b}{\partial x}\right|_{x=L_b} = 0.$$

At the interphase, two different boundary conditions must be satisfied: first, there must be equal fluxes from the textile for mass conservation; second, we will assume continuity of concentrations since the active principle is dissolved in serum at both sides. These conditions read:

$$-\phi S D \left.\frac{\partial c}{\partial x}\right|_{x=0^-} = -S_b D_b \left.\frac{\partial c_b}{\partial x}\right|_{x=0^+},$$
$$c(0,t) = c_b(0,t),$$

where ϕ is the porosity of the textile lattice, S is the area of the section of the textile film (m^2), and S_b is the area of the section of the Erlenmeyer flask (m^2).

A simple mass balance at $t \to \infty$ indicates that the total initial mass must be equal to the mass in each phase. Thus,

$$\phi S L c_0 = \phi S L c_\infty + S_b L_b c_\infty,$$
$$c_0 = c_\infty \left(1 + \frac{S_b L_b}{\phi S L}\right).$$

Perturbation Analysis

The analytical solution is achieved through an asymptotic expansion utilizing the small parameter $\delta = c_\infty / c_0 \ll 1$:

$$\hat{c} = \hat{c}^{(0)} + \delta \hat{c}^{(1)} + \delta^2 \hat{c}^{(2)} + O(\delta^3),$$
$$\hat{c}_b = \delta \hat{c}_b^{(1)} + \delta^2 \hat{c}_b^{(2)} + O(\delta^3),$$

where $\hat{c} = c/c_\infty$ and $\hat{c}_b = c_b/c_0$.

Applying Laplace transforms and residue theory to the leading order provides:

$$\hat{c}^{(0)} = \sum_{n=0}^{\infty} \frac{4 \sin\left(\frac{\pi(n+1/2)x}{L}\right)}{\pi(2n+1)} e^{-\frac{tD\pi^2(2n+1)^2}{4L^2}}.$$

First-order expansions correct the early bath dynamics, whereas second-order expansions capture remaining long-term concentrations within the textile substrate as $t \to \infty$.

Results and Verification

We verify the precision of the analytical perturbation expansions by comparing against numerical computations across both spatial profiles and time domains.

As shown in Figure 2, tracking the time profile at the extreme boundary coordinates ($x = -L$ and $x = L_b$) illustrates that the second-order perturbation expansion results match almost perfectly with the numerical solution due to the small parameter configuration ($\delta \ll 1$). Conversely, notice how a first-order solution deviates over long time scales because it fails to capture residual concentration storage trapped within the textile matrix. This behavior is also seen in Figures 3 and 4, which provide concentration profiles across the spatial domain at early and late intervals ($tD_b/L_b^2 = 0.03$ and $tD_b/L_b^2 = 0.5$).

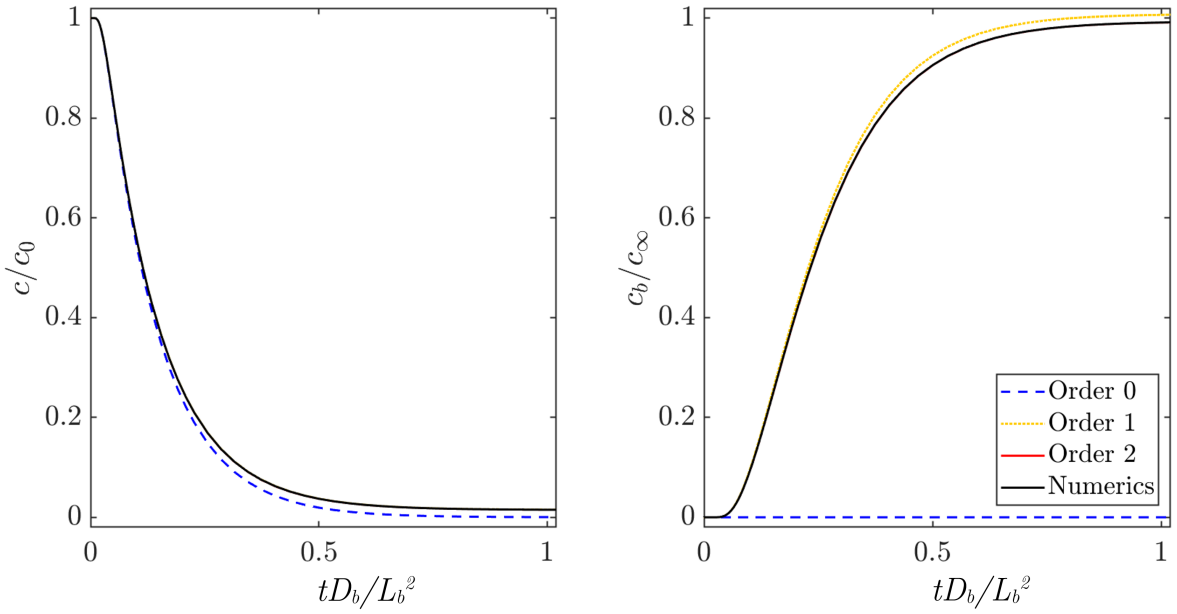


Fig. 2: Analytical vs numerical concentration responses over time t at the $x = -L$ and $x = L_b$.

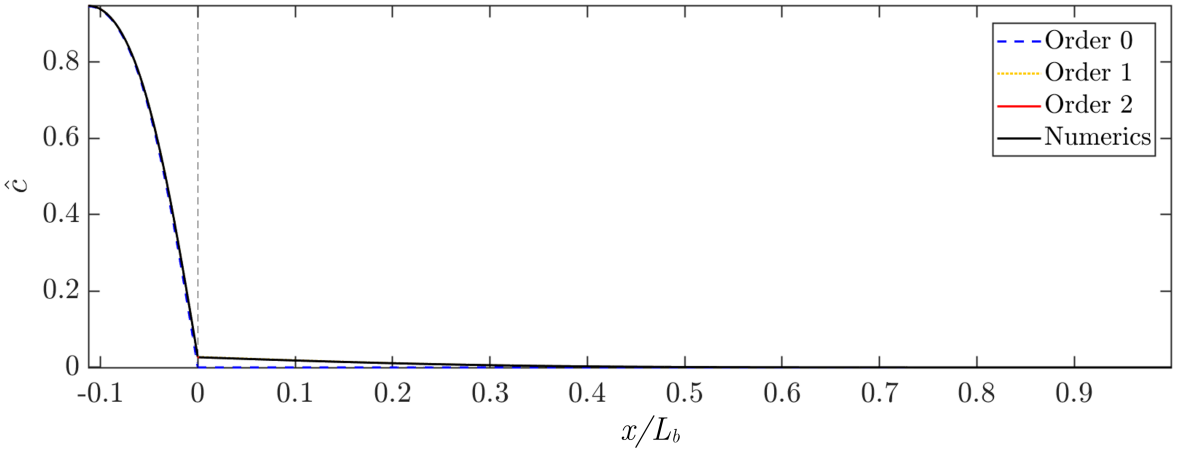


Fig. 3: Early-stage spatial validation ($tD_b/L_b^2 = 0.03$, i.e. 12 min).

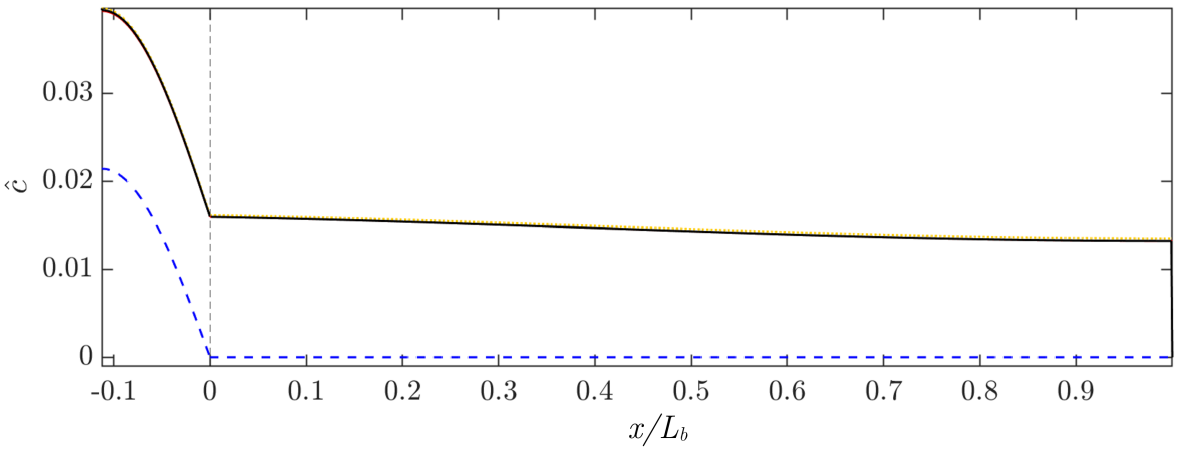


Fig. 4: Late-stage spatial validation ($tD_b/L_b^2 = 0.5$, i.e. 205 min).

The mathematical model was verified using laboratory measurements from lavender oil microencapsulation releases deployed inside cotton weave matrix trials.

Figure 5 highlights the concentration evolution inside the sweat-mimicking bath at $x = L_b/2$. The analytical solution closely reproduces the experimental observations by adjusting the effective diffusion in the microcapsule-textile media (D) and the diffusion coefficient in the bath (D_b).

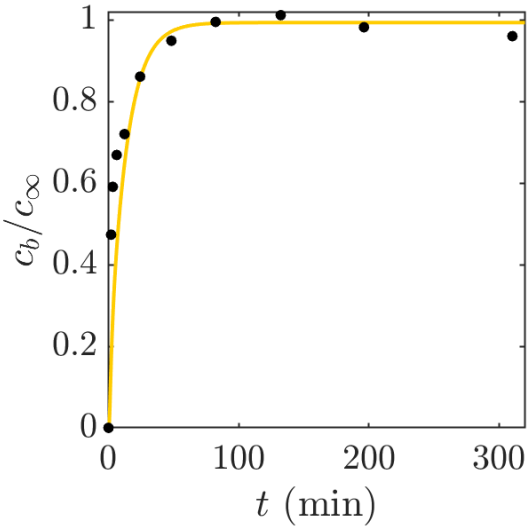


Fig. 5: Validation of analytical concentration profiles against experimental observations tracking lavender oil microcapsule active principle release into a serum bath.

Conclusions

A continuum physics-based model has been developed and validated to characterize active principle delivery within cosmetotextiles. Utilizing perturbation theory yields a computationally efficient and compact analytical solution capable of capturing the dynamic evolution of skin dosages. Future extensions of this work will incorporate the non-homogeneous capsule distributions typical of pad-dry-cure immobilization methods, which introduce an initial stepwise concentration profile. Furthermore, we intend to model active principle adsorption onto textile fibers, a phenomenon that generally induces a subtle decline in bath concentration driven by a backward concentration gradient.

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6

Educational Committee

News from the Educational Committee

During the year 2025, the ECMI Educational Committee resumed their regular activities.

The regular spring meeting was held at the Department of Mathematics of the University of Coimbra, Portugal, on March 14. Among other usual topics, during the meeting the members of the Educational Committee discussed about the ECMI Student Competition (ECMI SC). For the ECMI SC 2024 17 teams registered, but finally only one team submitted the final report. The Jury of ECMI SC 2024 decided to award the team from the University of Milan with the prize. However, the Committee started discussion about ideas on how to improve the performance of Student Competition.

At this meeting the Educational Committee voted in favour of accepting Técnico Lisboa/Instituto Superior Técnico as the ECMI Teaching Center within Mathematical Data Science and of accepting the University of Warsaw as the ECMI Teaching Center within Mathematics in Industry. The Committee decided on creating a document for inspected universities that clarifies how the process of accreditation is conducted, as well as guidance for the inspectors. There was also a discussion about changes to the rules for accreditation of the ECMI Teaching Centers in Mathematics for Industry Model Master. The initiative to create new rules for Mathematics for Industry Model Master stems from the fact that quite flexible, yet still quite demanding, rules for the Mathematical Data Science Model Master have recently been accepted by the Educational Committee. In recent years, we have observed programs changing; furthermore, master's programs in various European countries vary in length. All of this contributes to the need to update the rules for Mathematics for Industry and potentially unify them with the Mathematical Data Science rules.

The 38th ECMI Modelling Week, Kaunas, Lithuania



The 38th ECMI Modelling Week (ECMIMW2025) was successfully held at the Kaunas University of Technology from June 29 to July 05, 2025. This was the thirty eighth edition of the European Consortium for Mathematics in Industry (ECMI) an annual workshop designed for master students

from any field in mathematics. Organized under the umbrella of the ECMI, the event brought together over 50 master's students from across Europe. Over the course of one intensive week, participants worked in interdisciplinary teams to solve real-world industrial and business problems using mathematical tools. Participation in the event also contributes toward earning the ECMI Certificate – an internationally recognized qualification that attests to excellence in applied mathematics.

Each team, guided by an expert instructor, tackled a specific challenge introduced in daily language, which they then translated into a mathematical one. Working collaboratively, students developed and presented their solutions in scientific reports. This year's six modelling problems represented a wide range of fields. Group Testing in Large Populations (Adérito Araújo, Portugal): Students designed efficient testing strategies to identify infected individuals using minimal resources—vital for applications in epidemiology and logistics. Climate in the Light of Mathematical Equations (Davor Kumozec, Serbia): Using simplified differential models, students explored temperature change and climate sensitivity over time. Advancing Predictive Models for Blood Flow in Viscoelastic Vessels (Srdjan Lazendic, Belgium): This project applied fractional calculus to improve simulations of blood flow in flexible vessels, with relevance to medical diagnostics. Time and size of investments in green energy (Cláudia Nunes, Portugal): Students used stochastic modelling to determine when to invest in renewable energy under market uncertainty. Strategic placement of Wi-Fi routers (Kacper Taźbierski, Poland): Participants modeled electromagnetic wave propagation to optimize router positions for better signal coverage. Decoding Brain Activity (Karolina Armonaitė, Lithuania): Students employed neural networks and deep learning to interpret EEG signals and map brain activity.

Students' teams were formed to encourage cross-cultural and cross-disciplinary collaboration. Throughout the week, students worked independently with daily consultations and feedback sessions. Participants took part in evening events, including a mathematical treasure hunt across the KTU campus, a guided city tour of Kaunas, and a traditional Lithuanian food tasting session. The next ECMI modeling week will take place in Lund, Sweden.



During the Modeling Week, the Educational Committee held its regular meeting on Friday, July 4. The Committee members discussed the challenges associated with preparation and specifically that more than 20 mid- and large-scale companies had been asked for cooperation. None of the companies replied positively; they usually declare that they are interested but also state that they will not be able to create a working group to propose a mathematical problem. There was positive feedback from EU-MATHS-IN, which offered to contact companies that are gathered under the OpenDesk project. Further, a document named Instructions for MW instructors was prepared and approved by the members of Committee - it has been made available for all future organizers of Modelling weeks. The document will be incrementally updated if needed.

The autumn meeting was held online on 14 November. At the meeting it was again discussed about the Student Competition, where the 2025 problem was selected and that the competition website would be migrated to the ECMI blog. Additionally, the registration process was finalized and in future will most likely be managed via ECMI email servers. The upcoming Modelling Weeks in Lund (June 21-27, 2026) and Wuppertal in 2027 were also discussed. Finally, it was also decided to register a mini-symposium related to EduCom activities at the upcoming ECMI2026 conference.

Several more meetings were held before the end of the year, as in the meantime the company that was chosen to provide the SC problem decided to withdraw a few days before the announcement of the competition. Therefore, the Committee members started to consider other options, including providing an academic problem, i.e. a problem that would be proposed and provided by the ECMI Center, instead of a company. The final decision was to cancel the 2025 problem and focus on the following one for 2026.

Danijela Rajter-Ćirić

Chair of the Educational Committee

How to Profit from Uncertainty in the Green Energy Sector

Renewable energy is a rapidly expanding sector, strongly supported by governments worldwide. However, firms face the challenge of deciding *when* and *how much* to invest in such projects, in a context of strong price volatility, uncertain subsidies, and dynamic regulatory environments. This paper formulates the investment decision as an optimal stopping problem under uncertainty, and explores analytical, numerical, and simulation-based approaches to determine the best strategies for both firms and policymakers.

This problem was presented at the 38th ECMI Modeling Week, which took place between 29 June and 5 July 2025 at the Kaunas University of Technology, Lithuania. The report has been prepared by the students who worked on this problem under the supervision of Cláudia Nunes (Universidade de Lisboa, Portugal).

Problem Description

The renewable energy market is growing at an unprecedented pace. Its estimated value was 483.61 billion USD in 2024 and is projected to reach 1.2 trillion USD by 2035 [1]. For young firms entering this sector, investment decisions are often one-time and irreversible, which makes timing crucial. Prices, subsidies, and global conditions are highly uncertain and fluctuate on a daily basis. This situation naturally fits into the framework of an optimal stopping problem, where the challenge is to identify the optimal moment to invest in order to maximize expected profits.

The setup of the model assumes a constant investment cost I. Once the investment is made, the firm produces a fixed annual quantity Q. Electricity prices are modeled by a Geometric Brownian Motion (GBM):

$$dP_t = \mu P_t dt + \sigma P_t dB_t,$$

where P_t is the electricity price at time t, μ the drift under the risk-neutral measure, σ the volatility, and B_t a standard Brownian motion. The solution of this stochastic differential equation is

$$P_t = P_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma B_t},$$

which implies that electricity prices are lognormally distributed. Volatility broadens the distribution of possible future outcomes, reinforcing the relevance of optimal stopping strategies.

Profit Models with Subsidies

Government support plays a crucial role in shaping investment behavior. Two common types of subsidies are considered.

The first is a **fixed price subsidy**, where the firm receives a guaranteed price F_1 for each unit of electricity during the subsidy period:

$$\Pi_F(P_t) = \begin{cases} F_1 Q & \tau \le t \le \tau + T, \\ P_t Q & t > \tau + T. \end{cases}$$

The second is a **fixed premium subsidy**, where the market price is supplemented by an additional premium F_2 during the subsidy period:

$$\Pi_P(P_t) = \begin{cases} (P_t + F_2)Q & \tau \le t \le \tau + T, \\ P_t Q & t > \tau + T. \end{cases}$$

Here T denotes the subsidy duration and τ the investment time. These different schemes directly impact the investment threshold, and thus the timing of the decision.

The optimization problem consists in maximizing the expected net present value of profits:

$$W_S(P) = \sup_{\tau} \mathbb{E}\left[\int_{\tau}^{M} \Pi_S(P_t) e^{-rt} dt - I e^{-r\tau} \,\middle|\, P_0\right],$$

where M is the land lease duration, r the interest rate and S the subsidy type, fixed price (F) or premium (P) . The optimal strategy is typically of *threshold type*, meaning that the firm invests as soon as the price reaches or exceeds a critical value P^*.

Theoretical Formation

Analytical expressions for the investment thresholds can be derived [2]. For a fixed price subsidy, the threshold price is

$$P_F^* = \mathcal{A} \cdot \frac{I - F_1 Q \cdot \frac{1 - e^{-rT}}{r}}{e^{-(r-\mu)T}},$$

while for a fixed premium subsidy it is

$$P_P^* = \mathcal{A} \cdot \left(I - F_2 Q \cdot \frac{1 - e^{-rT}}{r}\right),$$

where

$$\mathcal{A} = \frac{\beta_1}{\beta_1 - 1} \cdot \frac{r - \mu}{Q}$$

and

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}.$$

From the government's perspective, it is important to design subsidy schemes that encourage investment while minimizing budgetary costs. A relation between F_1 and F_2 can be derived to ensure equivalent thresholds under both schemes ($P_F^* = P_P^*$):

$$F_2 = F_1 e^{(r-\mu)T} + \frac{I}{Q} \frac{r}{1 - e^{-rT}} \left(1 - e^{(r-\mu)T}\right).$$

Distribution of Investment Time

We assume that the electricity price follows a Geometric Brownian Motion (GBM):

$$dP_t = \mu P_t \, dt + \sigma \, P_t dB_t, \qquad P_0 > 0.$$

Taking logarithms of the explicit solution (6) gives

$$\ln P_t = \ln P_0 + \left(\mu - \tfrac{\sigma^2}{2}\right)t + \sigma B_t.$$

Define the normalized process

$$Y_t := \frac{\ln P_t - \ln P_0}{\sigma} \;=\; \bar{\mu}\, t + B_t,$$

with

$$\bar{\mu} := \mu - \frac{\sigma^2}{2}.$$

Let $P^* > P_0$ be the investment threshold, and set

$$a := \frac{\ln P^* - \ln P_0}{\sigma} > 0.$$

The optimal investment time, i.e., the first hitting time of the threshold, is

$$\tau \;=\; \inf\{t > 0 :\; P_t \ge P^*\} \;=\; \inf\{t > 0 :\; Y_t \ge a\}.$$

CDF via reflection principle with drift: For a Brownian motion with drift $W_t := B_t + \bar{\mu}t$, Girsanov's theorem together with the reflection principle gives, for $t > 0$,

$$\begin{aligned}\mathbb{P}(\tau \le t) &= \mathbb{P}\Big(\sup_{0\le s\le t} W_s \ge a\Big) \\ &= \Phi\left(\frac{\bar{\mu}t - a}{\sqrt{t}}\right) + e^{2\bar{\mu}a}\,\Phi\left(-\frac{\bar{\mu}t + a}{\sqrt{t}}\right),\end{aligned}$$

where Φ denotes the standard normal cumulative distribution function.

Density: Differentiating the CDF with respect to t, we obtain the density of the first-passage time:

$$f_\tau(t) = a\sqrt{\frac{1}{2\pi t^3}}\,\exp\left(-\frac{(-\bar{\mu}t + a)^2}{2t}\right), \qquad t > 0.$$

This is the density of an *Inverse Gaussian* distribution with parameters

$$\tau \sim \mathrm{IG}\left(\mu_{\mathrm{IG}} = \frac{a}{\bar{\mu}},\ \lambda_{\mathrm{IG}} = a^2\right), \quad \text{(for } \bar{\mu} > 0\text{)},$$

so that, for positive drift, the threshold is reached with probability one and $\mathbb{E}[\tau] = a/\bar{\mu}$. For $\bar{\mu} \le 0$, the threshold may never be reached and the density above describes the law of τ conditional on reaching the threshold.

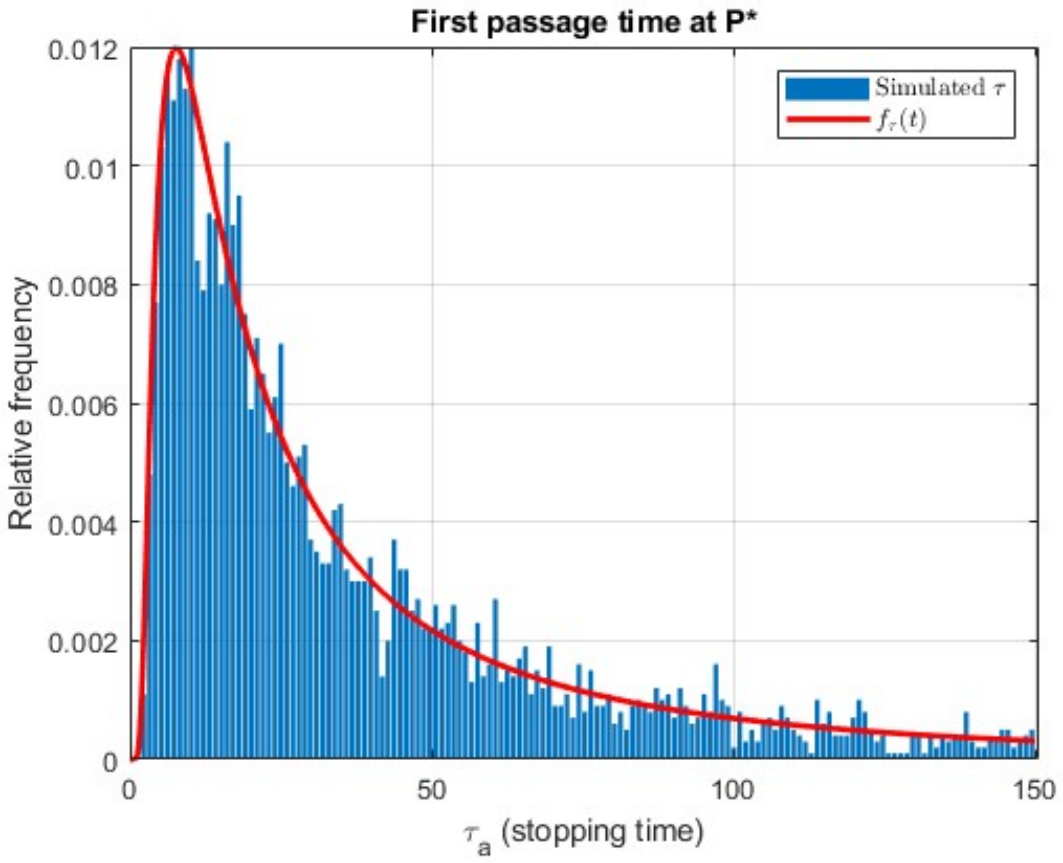


Fig. 1: Simulation of the stopping time τ and calculated theoretical density function.

Summary: With the notations introduced, the distribution of the optimal investment time τ can be expressed as

$$f_\tau(t) = a\sqrt{\frac{1}{2\pi t^3}}\,\exp\left(-\frac{(-\bar{\mu}t + a)^2}{2t}\right), \quad t > 0,$$

where

$$\bar{\mu} = \frac{\mu - \frac{\sigma^2}{2}}{\sigma}, \quad a = \frac{\ln(P^*) - \ln(P_0)}{\sigma}.$$

To validate the analytical expression, we also performed Monte Carlo simulations of electricity price paths, estimating the corresponding distribution of τ. The results are reported in Figure 1, which shows good agreement between the theoretical density and the simulated data.

Impact of Parameters on P^* and τ

In order to understand how model parameters influence the investment decision, we performed a series of sensitivity analyses, including risk-free rate, subsidy, its duration, etc. For example, Figures 2 and 3 illustrate how changes in the subsidy level F affect both the investment threshold P^* and the distribution of the stopping time τ. As expected, higher subsidies lower the critical price needed to trigger investment and shift the distribution of τ towards earlier times. Standard parameters used are found in Table 1. The bottom two images show the distribution of the stopping time τ for some chosen parameter values. The probability of never reaching the stopping time is printed for each value.

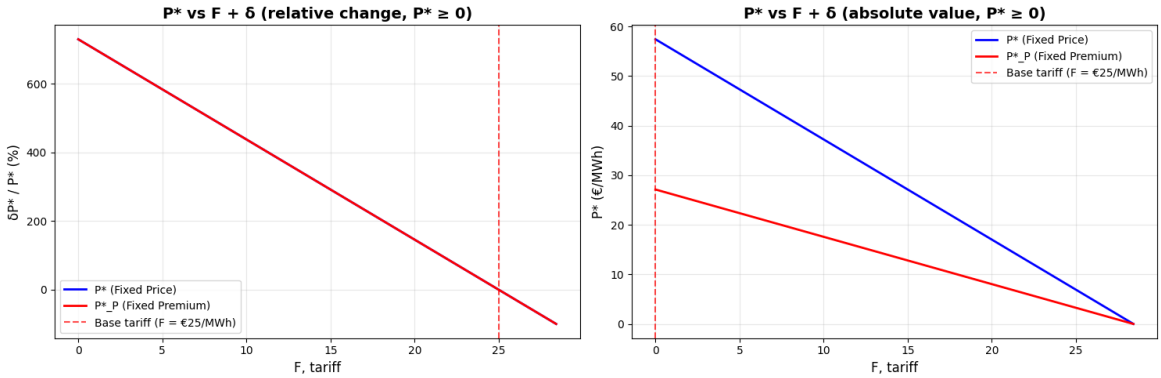


Fig. 2: Effect of changing F on the investment threshold P^*.

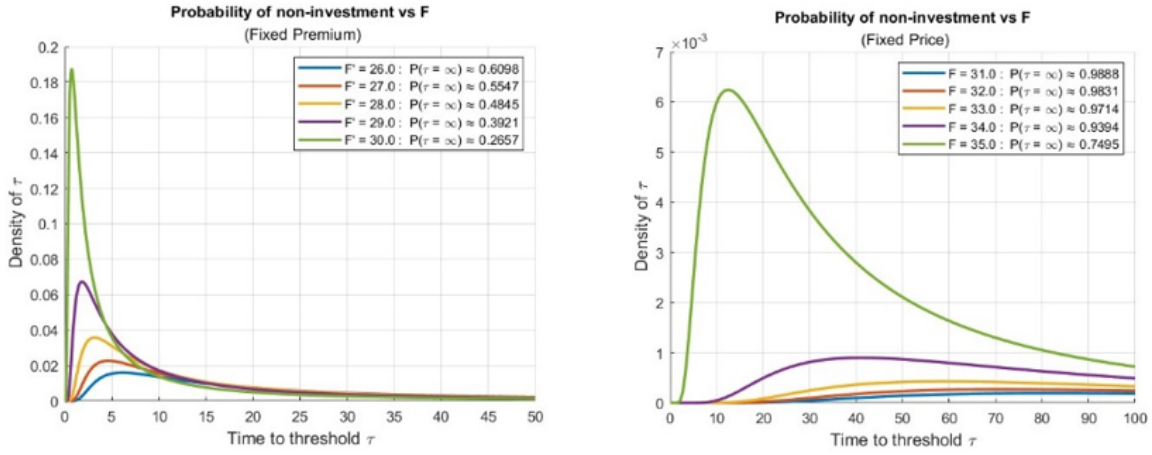


Fig. 3: Effect of changing F on the distribution of the stopping time τ.

Table 1: Standard model parameters used when calculating P_F^* and P_P^* in Figures 2 and 3.

Parameter	Value
r (risk-free rate)	5%
F (Subsidy)	€25/MWh
T (duration of subsidy)	15 years
μ (deterministic drift)	0%
σ (volatility)	19%
I (total investment cost)	€3 million
Q (annual produced quantity)	8000 MWh

Methods

To analyse the investment problem under uncertainty, we employ a combination of analytical, numerical, and simulation-based techniques. Monte Carlo simulations of electricity price paths are used to estimate the distribution of the investment time τ. The model is also extended to incorporate uncertain subsidy durations, modelled via exponential distributions, and solved using finite difference methods.

Analytical approximations provide tractable, although local, insights. For instance, linearization around a reference point yields

$$\tau = \frac{Q\mu + rI - rQP_0}{rQ\mu},$$

which gives a simple estimate of the optimal stopping time. However, such approximations are only reliable near the expansion point and under moderate volatility conditions.

Numerical methods, particularly backward induction combined with time–price discretization, are then used to solve the variational inequality associated with the value function. This approach produces time-dependent investment thresholds $P^*(t)$, which decrease as the deadline approaches, reflecting the urgency of investment under finite horizons. In contrast, for infinite horizons the threshold remains constant.

Analytical Approach

The analytical approximation is based on the linearization of the stochastic differential equations governing the dynamics of P_t, providing quick though limited insights into the timing problem. Its limitations lie in its validity only for small perturbations and short horizons.

The numerical approach, in contrast, is built upon the solution of a variational inequality. Time is discretized into N_t steps over $[0, M]$, and the price range is discretized into N_P grid points. Using backward induction, the algorithm starts at maturity $t = M$ with the boundary condition $V(M, P) = 0$. At each earlier time step, the method distinguishes between the continuation region, where the PDE

$$\partial_t V + \mu P \partial_P V + \tfrac{1}{2}\sigma^2 P^2 \partial_{PP} V - rV = 0$$

is solved numerically, and the stopping region, where the condition

$$V(t, P) = \Pi(P) - I$$

is enforced. This procedure yields the optimal stopping boundary $P^*(t)$ as a decreasing function of time, illustrating how finite horizons induce earlier investment decisions as maturity approaches.

Numerical Approach

To complement the analytical derivations, we implemented a Monte Carlo simulation approach to approximate the distribution of the optimal stopping time under different modelling assumptions. In this framework, a large number of sample paths of the electricity price process are generated, and the investment rule is applied to each trajectory to record the corresponding stopping time τ. Repeating this procedure allows us to estimate the empirical distribution of τ and compare it across alternative scenarios.

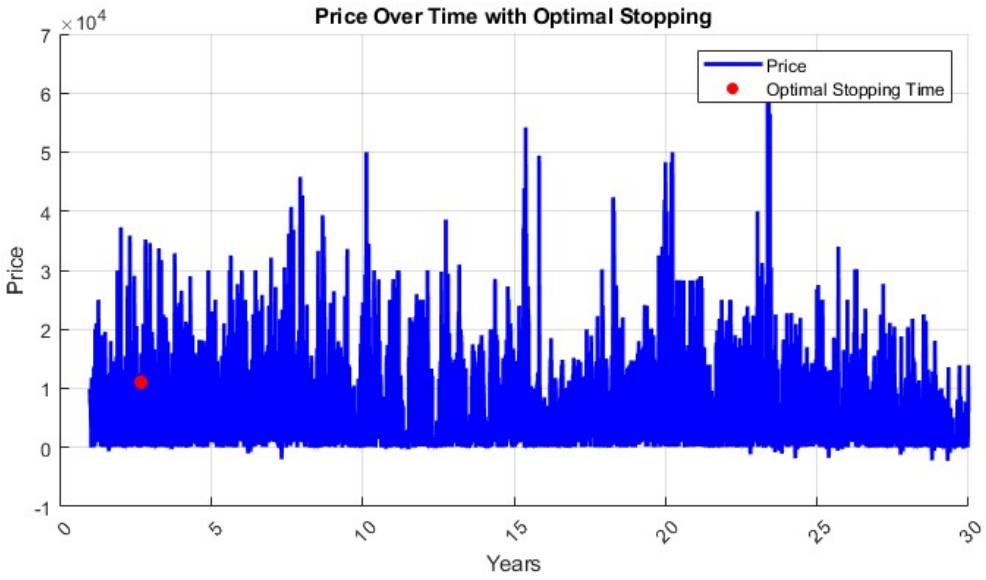


Fig. 4: Baseline simulation with deterministic production Q.

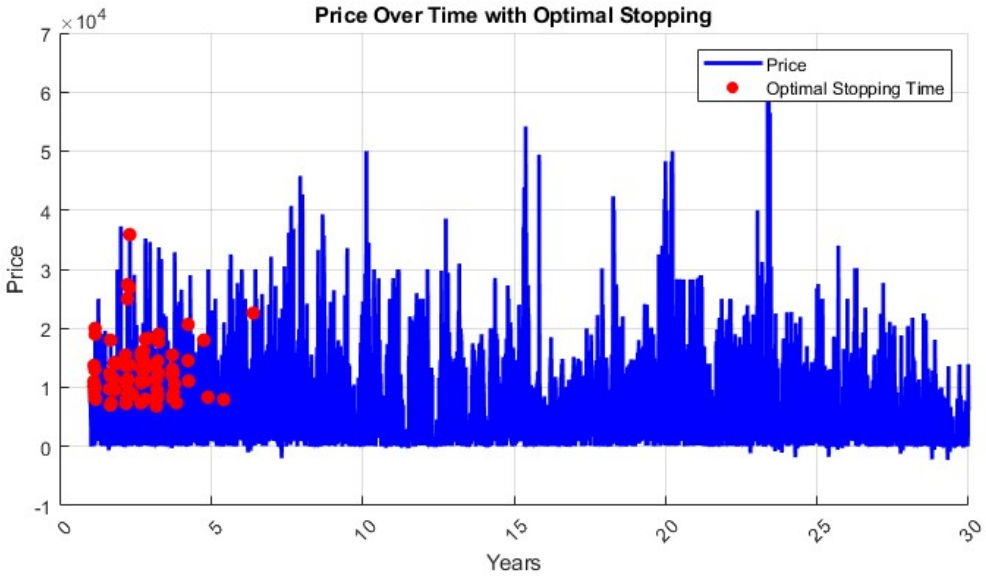


Fig. 5: Simulation with stochastic production Q, highlighting how randomness in output affects the shape and position of the investment threshold.

Overall, the Monte Carlo simulations confirm the theoretical insights while highlighting how different forms of uncertainty—whether in prices, production, or time horizon—shape the distribution of investment times.

Real World

Figures 4 and 5 present two examples based on real data. The first simulation (Figure 4) represents a baseline scenario with deterministic production Q, while the second (Figure 5) incorporates stochastic variations in Q. The results illustrate how parameter uncertainty influences both the shape and the position of the threshold boundary. Random fluctuations in production capacity broaden the range of potential investment times and shift the critical threshold, reflecting the increased risk faced by investors in more volatile environments. These findings underscore the importance of simulations in complementing analytical and numerical approaches, particularly when additional sources of uncertainty are considered.

Conclusion

Investments under uncertainty can be effectively addressed through optimal stopping theory and the use of threshold strategies. The design of subsidies plays a crucial role in determining both the optimal timing and the viability of projects, as fixed-price and fixed-premium schemes create markedly different dynamics. Incorporating finite horizons and regulatory risks adds realism to the model and motivates the use of numerical and simulation-based methods. Finally, transparency and policy stability remain key factors in building investor confidence and accelerating the adoption of green energy.

Future Work

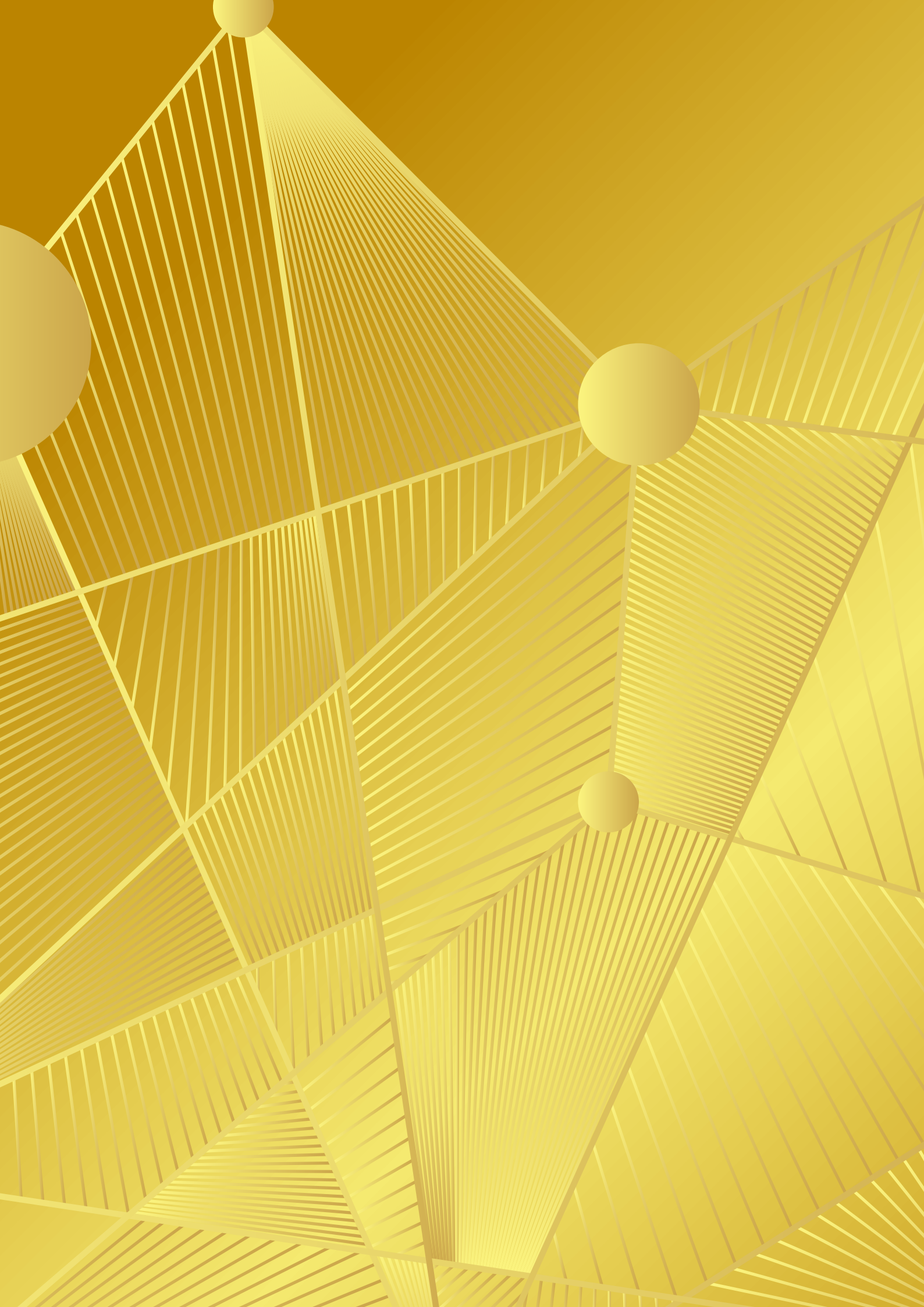
Future research may integrate machine learning approaches to better estimate price dynamics from real electricity market data. Adaptive subsidy mechanisms, incorporating feedback from market signals, represent another promising direction. Moreover, game-theoretic frameworks and multi-agent simulations could capture competition among firms and co-optimization between governments and private actors. Finally, incorporating carbon credit pricing and storage technologies will enrich the model and provide more comprehensive insights.

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7

ECMI Special Interest Groups

Introduction

Special Interest Groups (SIGs) exist to promote collaborative research on specific topics in Mathematics for Industry within Europe. A particular aim is to enable researchers from both academia and industry with similar interests to get together and submit proposals for funding to the European Union or to other funding bodies. ECMI can act as a catalyst in the formation of such a group by offering advice about the expertise available within Europe, by posting information on the web pages, and by circulating information about events to all members.

Computational Finance and Energy Markets

Chairs: Matthias Ehrhardt (University of Wuppertal, Germany), Jan ter Maten (University of Wuppertal, Germany).
Scope: The objective of this SIG is to gather specialists from all over Europe to tackle the actual problems of the modelling and the numerical analysis of various problems in computational finance. We aim to create a network of experts in the field to make communication easier, and to enable industrial or governmental organizations to find the right expert for their needs. The network is truly multidisciplinary, combining the power of mathematics, scientific computing, and quantitative finance, for modelling, calibration and simulation.

Math for the Digital Factory

Chairs: Dietmar Hömberg (Weierstrass Institute for Applied Analysis and Stochastics, Germany), Joachim Linn (Fraunhofer Institute for Industrial Mathematics, Germany).
Scope: The term Digital Factory represents the shift in industrial manufacturing towards the integration of all processes in the industrial value chain, sometimes also referred to as the 4th industrial revolution - Industry 4.0 (in Europe) or Smart Manufacturing (in the United States). The SIG's main goals are:

1. Reconfigurable, adaptive and evolving factories capable of small scale production
2. High performance production, combining flexibility, productivity, precision and zero defects
3. Energy and resource efficiency in manufacturing

None of these goals can be achieved without thorough modelling of all aspects of manufacturing together with a multi-scale simulation and optimization of process chains, i.e., these goals cannot be achieved without mathematics.

MaDiFa brings together university mathematicians working in modelling, simulation and optimization related to manufacturing with practitioners from manufacturing industry. The general scientific goal is to develop a holistic mathematical view on digital manufacturing.

Mathematics for Big Data and AI

Chairs: Natasa Krejic (University of Novi Sad, Serbia), Alessandra Micheletti (University of Milan, Italy).
Scope: Sometimes data are "big" because of their high dimensionality and space-time structure, e.g., satellite images, signals registered by sensors or antennas. In such cases suitable mathematical techniques for dimensionality reduction are needed, both for data visualization, and for their numerical treatment. Functional Statistics, that is a field in which a

lot of research is concentrating nowadays, may help in facing this task. In other contexts data are considered “big” because of their complexity or heterogeneity, e.g., data extracted from social networks with text mining, mixed with socioeconomic data for marketing purposes; or data highly interrelated which may be represented by complex graphs, like atoms and bounds in a protein, or relationships between users of a social network. Sentiment analysis and Topological Data Analysis are new statistical fields of research, still under development, which may help to tackle the problem of analysing such data. The aim of this SIG is to collect people working on the themes described above, coming both from academy, and from “industry” (to be intended in a wide sense) to favour scientific collaboration and research.

Modelling, Simulation and Optimization in Electrical Engineering (MSOEE)

Chairs: Stefan Kurz (Robert Bosch GmbH, Germany), Sebastian Schöps (TU Darmstadt, Germany).
Scope: Electrical engineering is an important technology for many recent societal and industrial developments. It includes the investigation and application of electricity, electronics, and electromagnetism. For example, smartphones are connected to the internet by using semiconductors based on nanometer technology for processing data and at the same time antennas for exchanging data with cell towers which are tens of kilometers away. More generally, equipment for mobile communication is used to control many other applications, e.g. in health care, banking, security, autonomous driving or energy distribution. Particularly, the transition towards sustainable energy requires the improvement of electrical infrastructure, again, from small scale, e.g. household devices, electric machines, up to power grids. MSO of electric machines is of key importance for electric vehicle applications. The importance of the underlying technologies is also recognized by the European Union in various European Technology Platforms, e.g. Electronic Components and Systems for European Leadership or Smart Networks for Energy Transition.

Mathematical challenges in electrical engineering are often driven by industrial needs and are related to classical and new emerging topics of applied mathematics and scientific computing, i.e., modeling, simulation, optimization, uncertainty quantification, stochastics and data analysis. Thus, questions of existence and uniqueness related to new semiconductor models are as much relevant for this interest group as the development of new and efficient simulation techniques for electrical machinery. Recent examples of European and domestic research efforts in that field are ASIVA14, nanoCOPS and PASIROM.

Sustainable Energies

Chairs: Nella Rotundo (U. Firenze, Italy), Ferran Brosa Planella (U. Warwick, UK), Dirk Peschka (Weierstrass Institute, Germany), Timothy G. Myers (Centre de Recerca Matemàtica, Spain).
Scope: The transition to a sustainable future is one of the most important goals of society, as clearly identified by the UN Sustainable Development Goals (SDGs). This process is currently influencing scientific, industrial and political developments to a great extent. In particular, the development of sustainable energies and materials are two areas with a very high impact on industry and which can contribute to several SDGs.

The goal of this SIG is to continuously improve the processes involved in the energy and material industry, to reduce their cost and their environmental impact. For energy, for example, this involves replacing limited or polluting energy sources with renewable and environmentally friendly ones, improving the efficiency of the distribution networks and keeping in mind the

stability of the energy supply. For materials, this involves reducing the use of energy and scarce raw materials in the production processes, and improving the recycling processes.

Our aim is to transfer fundamental mathematical methods for Modelling, Simulation, Optimization (MSO) to the fields of energy generation, transport and storage, material extraction, production and recycling. At present the focus of the SIG are MSO of:

- solar panels, wind turbines and beyond,
- model hierarchies for energy networks,
- batteries,
- carbon capture and contaminant removal,

but we are open to new topics and mathematical aspects.

Computational Physiology and Mathematical Epidemiology

Chairs: Thomas Götz (U. Knoblenz, Germany), Giovanni Naldi (U Milan).
Scope: The primary scope of this SIG centers around Biomathematics, with a core focus on healthcare, health, medicine, and biosciences. It actively explores the intersection of these fields with Explainable, Trustworthy, and Robust Artificial Intelligence (XAI), mathematical and computational modeling, and a human-centered perspective on AI interpretability. A key objective is establishing a broad collaborative network of ECMI and non-ECMI members, hospitals, research centers, and industry partners to layout groundwork for future funded research projects.

To date, the group has focused on the following activities:

- Network Building: Successfully brought together interest and collaborations from researchers across multiple international institutions, including Sorbonne-Université, Università di Genova, Bocconi University / Duke University, Università di Pavia, Università di Milano, and the University of Warwick.
- Event Planning: Initiated preparations for an upcoming inaugural meeting, targeted for ECMI 2026.
- Mini-Workshops: Co-organized a dedicated mini-workshop alongside the DESIRE Center focusing on health and healthcare topics, aimed at assessing engagement and involving clinical and research institutions.
- Educational Initiatives & Summer Schools: Organized the ExTRAInt doctoral summer school in Como, Italy (September 8–11, 2025), which was financially supported by ECMI. The school trained PhD students and young researchers on advanced XAI methods, geometric/analytical robustness in AI, green computing, neuroscience, and medical decision-making. This initiative blended academic student seminars with practical insights from industry and consulting experts.

8

European Study Groups with Industry

European Study Groups with Industry

The three pillars of ECMI are research, education and outreach and ESGIs are a key vehicle for all of these. Study Groups are an internationally recognised method of technology transfer between mathematicians and industry and have led to a multitude of research projects and novel areas of research.

Initiated in Oxford in 1968 they were originally termed the Oxford Study Groups with Industry and for 30 years remained a UK activity. The natural progression to a European activity, under the aegis of ECMI, occurred in 1998 when the first meeting was held outside of the UK, in Lyngby. Perversely this was denoted the 32nd European Study Group with Industry (presumably to avoid the confusion caused by the Oxford Study Groups moving around UK universities). By the end of 2024 we had celebrated the 185th meeting. The format has proved so popular that similar meetings are now held around the world.

By 2014 there was an average of 9 to 10 ESGIs per year, this rate slowed down during the COVID period, with a low of 3 meetings in 2020. This has rapidly improved, with 6 in 2024 and 8 in 2025, this includes a study group focussed on Processing and Safety in Fluids, held in Leeds and described later in this section.

ECMI members have also actively participated in international study groups. Annual Meetings are held in Canada (Industrial Problem Solving Workshop), Australia (Mathematics in Industry Study Group), South Africa (also MISG) and the US (Mathematical Problems in Industry Workshop). Most of these are announced on our International Study Groups page on the ECMI website (if I have missed any, please contact me and I can add a link to the website).

The list of upcoming and past ESGIs may be found on the ECMI Blog at

https://ecmiindmath.org/european-study-groups-with-industry/

International meetings are listed at

https://ecmiindmath.org/international-study-groups-with-industry/

UK study groups now have a dedicated website:

https://www.esgiuk.org.uk/

(perhaps they are considering an ESGI Brexit?). I should also mention that the 200th ESGI will be held at Warwick University in 2027. I was a main organiser of the 29th meeting held in Oxford in 1996 - I feel very old.

Information on the Dutch meetings back to 1998 may be found at

https://www.swi-wiskunde.nl/

A key resource is the Maths in Industry Repository (MiIR) - a website with reports from study groups throughout the world. It is sponsored by ECMI and hosted by Cambridge University Press,

www.cambridge.org/engage/miir

For any advice, the ESGI Co-ordinator (and overworked Annual Report editor) is occasionally willing to help: Prof. Tim Myers.

Timothy G. Myers
Centre de Recerca Matemàtica, Spain
coordinator_esgi@ecmi-indmath.org.

Preparing Crypto Market Data for Analysis and High Frequency Decision Making

This study develops an analytical framework for processing ultra–high-frequency cryptocurrency market data. The focus is on transforming raw trade and order-book events into regular time-series suitable for statistical modelling and machine learning; building a historical evaluation module; and developing forecasting and risk estimation models. The results show that cryptocurrency markets exhibit strong time-scale dependency. Liquidity analysis confirms that Bitcoin is significantly more liquid than smaller assets, as reflected by tighter bid-ask spreads. Risk-adjusted performance measures are found to be highly sensitive to the chosen sampling frequency. Overall, the findings demonstrate that robust analysis of cryptocurrency markets requires combining distributional properties of returns.

This problem was presented at ESGI 189, held in Kaunas (Lithuania) from June 9 to June 13 2025.

Problem description

Preparing microsecond-level cryptocurrency market data for analysis and high-frequency decision making: transforming raw order-book and trade events into regular time-series suitable for statistical modelling and machine learning; building a historical evaluation module; and developing forecasting and risk estimation models.

The primary objective of this study is to develop and evaluate a comprehensive analytical framework for processing ultra–high-frequency cryptocurrency market data and extracting meaningful information for forecasting, risk assessment, and decision support.

The specific objectives of the research are as follows:

- to prepare and structure high-frequency trade and order-book data in a form suitable for statistical analysis and machine learning applications;
- to analyse the relationship between transaction prices and market microstructure variables, in particular the bid–ask midpoint and spread;
- to investigate the statistical properties of high-frequency cryptocurrency returns, volumes, and inter-event durations, with special attention to non-Gaussian behaviour and heavy tails;
- to evaluate volatility dynamics and liquidity risk using both classical and tail-risk-based measures;
- to assess the performance of selected indicators and models using risk-adjusted performance metrics;
- to compare results across different assets and temporal resolutions (1-second and 1-minute data);
- to provide empirical insights relevant for high-frequency trading, monitoring, and anomaly detection in cryptocurrency markets.

Challenges

Key challenges include data (see Figure 1) volume and irregular event timing, low-activity intervals, microstructure noise (bid–ask, spread, slippage), and heavy-tailed non-Gaussian distributions of returns/volumes/durations that require robust modelling and risk-aware evaluation. The analyst in this case must be aware of memory overload for data storage, as there might be thousands of active ask/bid orders.

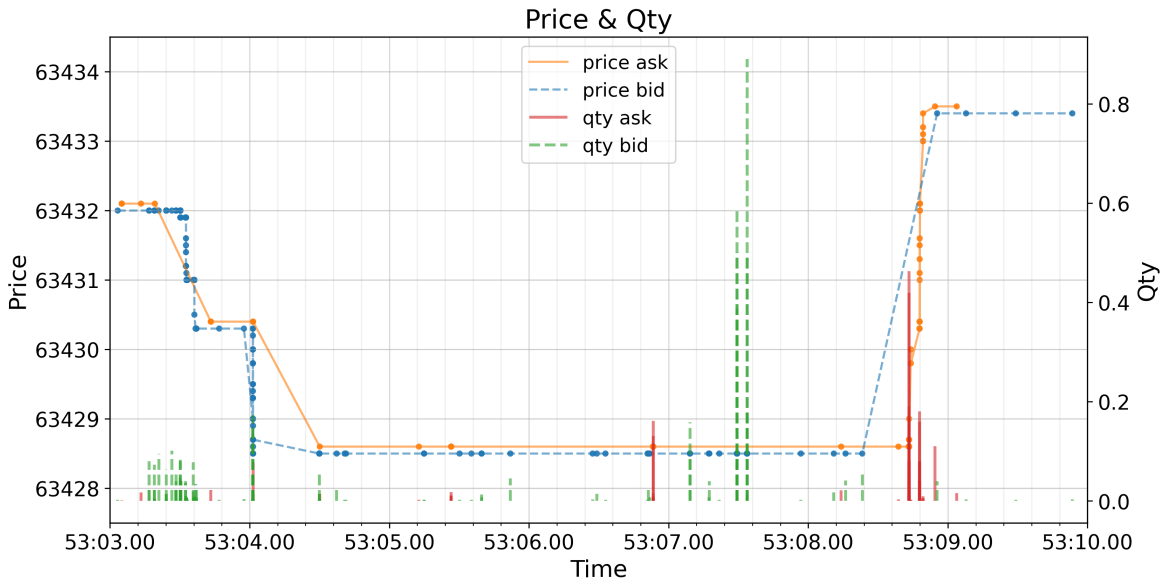


Fig. 1: Best ask & best bid prices of BTC within 7 seconds of trade on 01-10-2024.

Research Methodology

This study applies an empirical methodology for processing and analysing ultra–high-frequency cryptocurrency market data with the aim of examining market microstructure, return behaviour, liquidity, and risk–return characteristics.

The analysis is based on raw trade and order-book data obtained at microsecond resolution. Since market events occur at irregular time intervals, the data are transformed into regular time series using fixed sampling grids. Two aggregation frequencies are used throughout the study: 1-second and 1-minute intervals, which allow comparison across different time scales and reduce microstructure noise [1].

Several price representations are considered. In addition to transaction prices, the bid–ask midpoint is calculated as the arithmetic mean of the best bid and best ask quotes. This variable is used as a proxy for the underlying market price in high-frequency settings, while the bid–ask spread is analysed as an indicator of liquidity conditions and market frictions.

Market activity is evaluated through waiting times between consecutive transactions or order-book updates. Their empirical distributions are analysed and compared with theoretical distributions in order to identify suitable models for inter-event durations.

Return series are computed at both sampling frequencies. Due to the high proportion of zero returns in ultra–high-frequency data, zero and non-zero observations are analysed separately. The distributional properties of returns are examined with particular attention to heavy tails and non-Gaussian behaviour, which are well documented in cryptocurrency markets [2].

Liquidity and risk are evaluated using both spread-based and return-based measures. The analysis includes Value at Risk (VaR), Conditional Value at Risk (CVaR), downside-sensitive metrics, maximum drawdown, and the Amihud illiquidity measure [3]. These indicators allow comparison of risk and liquidity across assets and time resolutions.

To assess practical relevance, risk-adjusted performance measures are computed, including the Sharpe, Sortino, Calmar, Rachev, Omega, Pain, and Gain/Loss ratios. These metrics are used to evaluate performance under different market conditions and sampling frequencies.

Overall, the methodology is designed to ensure consistency between the applied methods and the empirical results presented in subsequent sections.

Research Results

High-frequency cryptocurrency markets are characterised by extremely large data volumes, irregular event timing, and rapid price fluctuations. Trading activity may occur at sub-second intervals, generating large datasets and requiring specialised data processing approaches [4].

The empirical analysis confirms that the bid–ask midpoint provides a highly accurate approximation of transaction prices at ultra–high frequencies. The deviation between midpoint and executed prices remains negligible, which makes the midpoint a reliable proxy for the underlying market price and suitable for short-horizon modelling and forecasting [5].

Return dynamics exhibit strong deviations from Gaussian assumptions. At high sampling frequencies, a substantial proportion of observations correspond to zero returns, reflecting periods of no price

change. For non-zero returns, the distributions display heavy tails and skewness, indicating a high probability of extreme price movements, which is consistent with previous findings in cryptocurrency markets [2].

Liquidity conditions vary over time and are closely related to price dynamics. The bid–ask spread demonstrates time-varying behaviour and tends to increase during periods of market stress and heightened uncertainty. Changes in spread and trading activity may precede significant price movements, suggesting their usefulness as early indicators of market instability.

Risk-adjusted performance measures are found to be highly sensitive to the chosen sampling frequency. Metrics computed at lower frequencies may overestimate performance, while high-frequency evaluation reveals greater variability and exposure to microstructure effects and transaction costs.

These findings highlight the importance of combining appropriate data representation, distributional modelling, and liquidity-aware risk evaluation when analysing high-frequency cryptocurrency markets.

Waiting time between transactions

In this study, data from the Binance cryptocurrency exchange are used at a one-microsecond frequency. This is a standard data frequency; therefore, changing it requires special functions and the selection of an aggregation method. The data can be aggregated into regular intervals (e.g., 1-second or 1-minute intervals) to enable the application of time series models that allow the analysis of frequency and periodicity across different time scales [1]. The analysis of higher-resolution data makes it possible to examine market microstructure in greater detail. Aggregation helps transform high-frequency data into lower-frequency data, i.e., instead of analyzing second-level data, one can analyze hourly, minute-level, or even daily intervals. When aggregating data over a selected period (e.g., 5 minutes or 1 hour):

1. The following prices are most commonly calculated:
   - Opening price (open) – the price of the first trade in the period;
   - Closing price (close) – the price of the last trade in the period;
   - Highest (high) and lowest (low) prices within the interval;
2. Volume: all traded volumes within the selected period are summed;
3. Number of trades: the total number of trades in the aggregated interval is counted.

For example, if tick-by-tick data are available, they can be aggregated into 5-minute candles (so-called OHLC bars), thereby reducing the data volume while preserving the essential information. The simplest aggregation method is previous-tick interpolation [6].

The next step is the selection of the trading period. This is the time interval used in the analysis on which the trading frequency is calculated. The choice of period depends on the research objective: high-frequency analysis requires short intervals (e.g., 1 minute or even seconds), while longer-term analysis may use daily, weekly, or monthly intervals. The researcher must decide which periods are most appropriate for observing the phenomenon under study, since different periods highlight different aspects of market behavior [7].

Trading frequency is calculated as the number of trades per selected unit of time. This can be done by counting how many trades occurred in each time period and then computing the average. For example, if 1,000 trades occur during a trading day and

the trading day lasts 6.5 hours, the average trading frequency is approximately 154 trades per hour. It is also possible to analyze the distribution of trading frequency: identifying peak periods, comparing trading intensity across different hours of the day or different days of the week.

The histogram of time differences and the corresponding approximating curves of the fitted distributions (log-normal, exponential, and gamma distributions with their respective parameters) are shown in Figure 2.

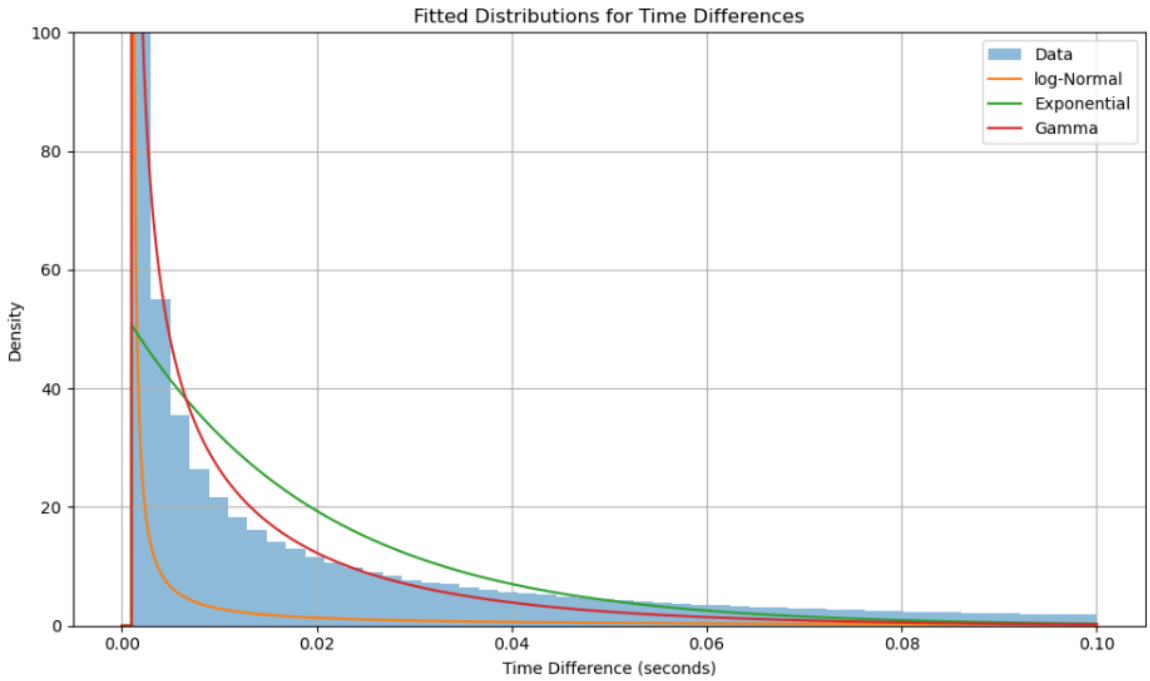


Fig. 2: Relationship between waiting time (s) and frequency (count) between two recorded ask/bid price entries in the order book on 2024-10-01.

Table 1: Results of goodness-of-fit tests for waiting-time distributions.

Distribution	Log-Likelihood	BIC	KS Statistic	KS p-value
lognorm	7.88E+06	-1.58E+07	0.443404	0
expon	3.52E+06	-7.03E+06	0.206524	0
gamma	6.93E+06	-1.39E+07	0.160163	0
beta	6.49E+06	-1.30E+07	0.181037	0

According to the KS statistic (Table 1), the gamma distribution provides the best fit for the time intervals between non-zero waiting times.

However, it should be noted that BTC is a highly liquid asset; therefore, this pattern may differ for other cryptocurrencies.

Such visualizations allow the identification of activity trends, such as increased trading frequency during market opening or closing periods. Moving averages can also be applied to reduce data noise and make trends more clearly visible [8].

Although large trading gaps are not observed in the analyzed datasets, low-activity periods are present, particularly for lower-liquidity cryptocurrencies, where inter-transaction waiting times increase.

Analysis of General Characteristics of Financial Instruments in Trading Markets

Cryptocurrency markets exhibit distinctive characteristics compared to traditional financial assets, particularly in terms of volatility, liquidity, and trading intensity. High-frequency data reveal rapid price changes, irregular trading activity, and strong deviations from standard statistical assumptions.

Empirical evidence shows that return distributions are typically non-Gaussian, exhibiting skewness and heavy tails, which implies a higher probability of extreme price movements [9]. These effects become more pronounced at higher sampling frequencies, where market microstructure plays a significant role.

Liquidity conditions vary across assets and time periods. Highly liquid instruments such as Bitcoin tend to have narrower bid–ask spreads and more frequent trading, while less liquid assets exhibit wider spreads and more irregular price dynamics [10].

Trading volume and volatility are closely related. Periods of increased trading activity are often associated with heightened volatility, reflecting intensified information flow and market reactions. At the same time, the choice of sampling frequency significantly influences observed statistical properties, as lower-frequency aggregation may mask short-term fluctuations.

These general characteristics provide the foundation for analysing short-term price dynamics and fluctuations in cryptocurrency markets.

Price fluctuations in the market

Cryptocurrency prices are highly volatile and exhibit frequent and sometimes abrupt fluctuations driven by market dynamics, external information, and speculative activity. Compared to traditional financial assets, price changes are more extreme and occur over much shorter time intervals [11]. Numerous empirical studies show that cryptocurrency returns are characterised by heavy-tailed distributions, meaning that large price changes occur more frequently than under Gaussian assumptions. This property makes price dynamics difficult to model using classical statistical approaches and requires more flexible modelling frameworks. At ultra–high frequencies, an additional feature emerges: a large proportion of observations correspond to zero returns [6], reflecting periods with no price change. In the analysed dataset, zero returns account for a significant share of all observations, indicating that price dynamics at very fine time scales are dominated by small or null movements.

After separating zero returns, non-zero returns can be analysed using continuous distributions. In this study, alpha-stable distributions are used to capture heavy tails

and skewness in return behaviour, providing a more adequate representation of extreme price fluctuations.

Figure 3 presents the empirical distribution of logarithmic returns together with the fitted alpha-stable density.

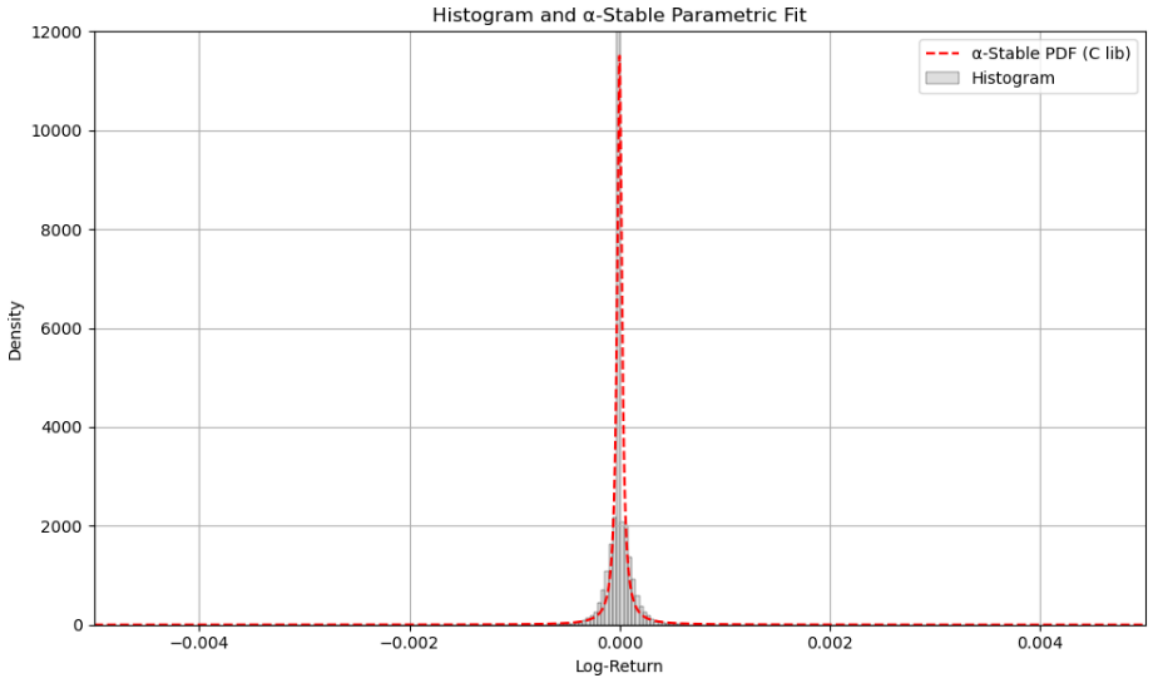


Fig. 3: BTC (1s) logarithmic returns distribution (non-zero returns) with fitted alpha-stable law.

It turns out that the number of zero returns is exceptionally high: 27,405 out of 86,397, which constitutes 31.71% of all returns. Therefore, models described in Kabasinskas et al. [6] can be applied for return modelling. In this case, after separating zero returns, non-zero returns can be treated as drawn from one of the continuous laws (e.g., normal, alpha-stable, etc.). Figure 3 shows the histogram of non-zero returns and the alpha-stable density with parameters $(\alpha = 0.9987, \beta = 0.0194, \sigma = 0.00002766, \mu_0 = -0.00000177)$.

From the parameter values one can observe that $\alpha \approx 1$, which theoretically implies that the efficient market hypothesis does not hold for this cryptocurrency and that forecasting its returns is not possible at 1s resolution.

Changing the data resolution yields somewhat different results. Using 1-minute frequency gives parameters $\alpha = 1.4368$, $\beta = -0.0254$, $\sigma = 0.00038147$, $\mu_0 = -0.00001724$, which differs strongly from the 1s results. The BTC (1min) distribution of logarithmic returns (after removing zero returns) and the fitted alpha-stable law are shown in Figure 4.

Another important aspect: different time periods also yield different results. For example, in a later period, 1s resolution data contain considerably more zero returns. Most likely, to forecast price changes for highly liquid cryptocurrencies and for other ones, different mathematical models will need to be developed and different machine learning architectures will need to be trained.

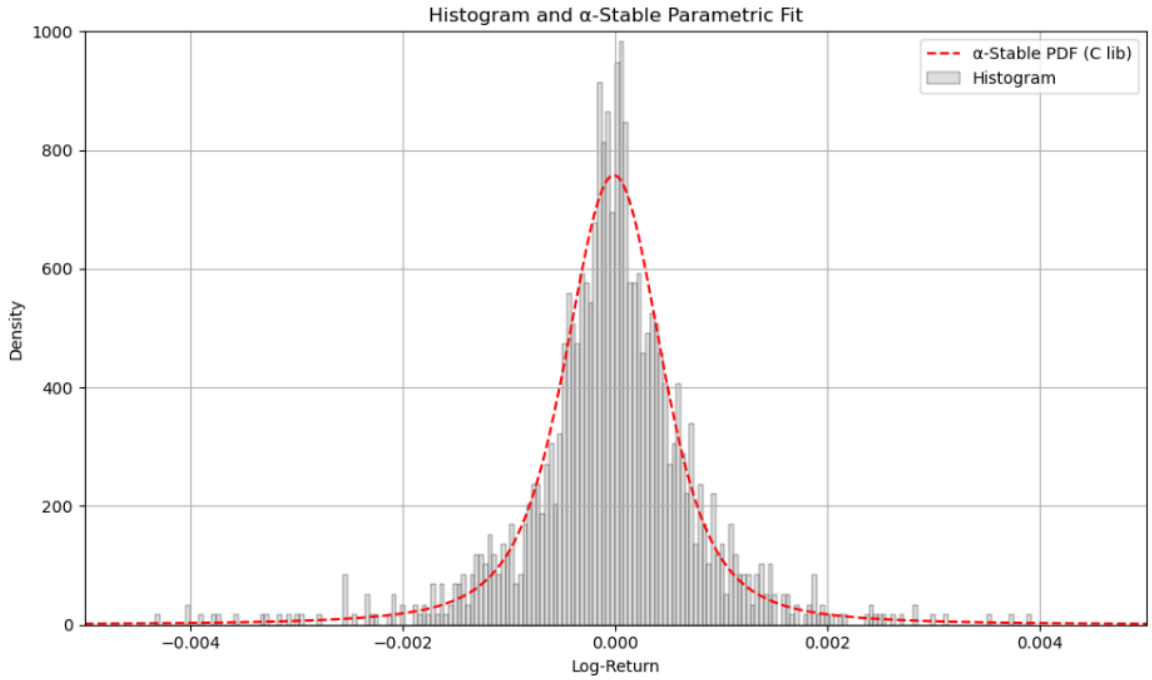


Fig. 4: BTC (1min) distribution of logarithmic returns on 2024-10-01 (non-zero returns) with fitted alpha-stable law.

Typical trading volumes, turnover, and their outliers

Trading volumes in high-frequency cryptocurrency markets vary substantially over time and often exhibit extreme values. Empirical analysis shows that turnover tends to increase during periods of strong price movements, indicating a close relationship between market activity and return dynamics [10].

It turns out there are certain days when turnover increases very strongly. For example, on 2025-05-21 turnover jumped sharply at around 21:00 Lithuanian time, when the US President met with the President of South Africa. Although these events are not necessarily related, this could be an example of information spillover.

Quite often, a tendency is observed that turnover increases when the price changes strongly. Figure 5 presents the relationship between turnover and returns. The results show that large turnover values (exceeding 2–3 standard deviations) are typically associated with similarly large price changes. This suggests that periods of elevated trading activity coincide with heightened volatility and intensified market participation.

Such behaviour indicates the presence of heavy-tailed dynamics, where extreme observations occur more frequently than under normal distribution assumptions. These findings are consistent with the previously observed non-Gaussian return structure and reinforce the importance of using robust statistical models in high-frequency market analysis. This effect is particularly pronounced at higher frequencies, where market reactions to new information are immediate.

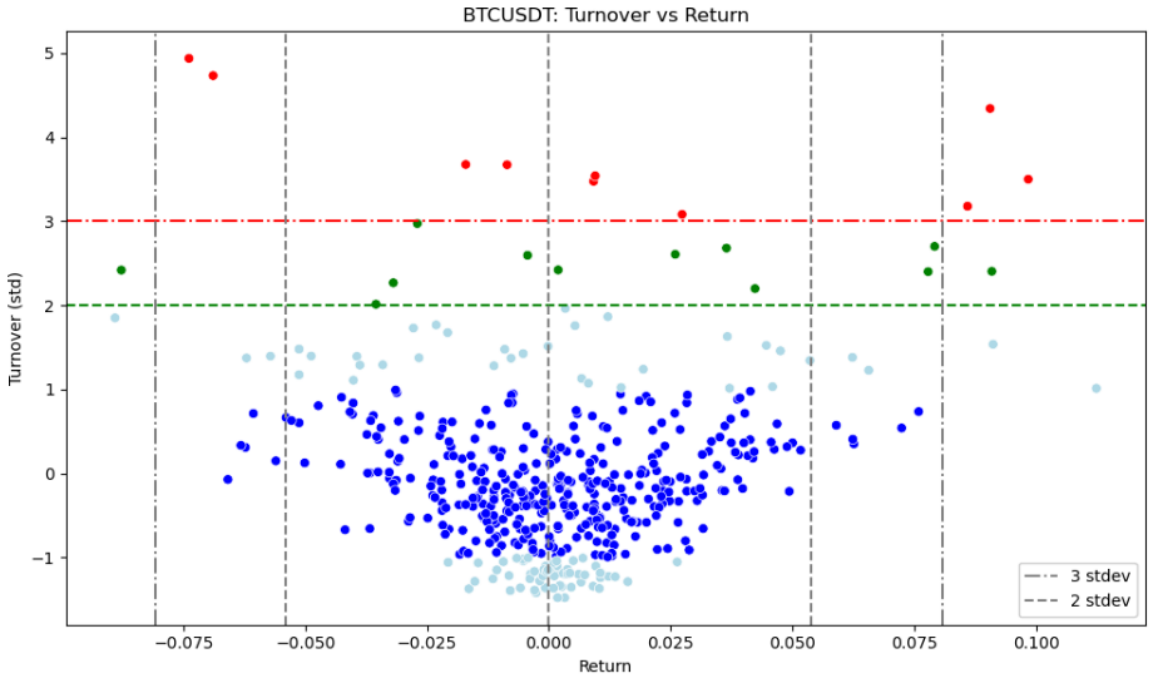


Fig. 5: Turnover dependence on return.

Risk–return and liquidity metrics

Modern Portfolio Theory highlights the importance of the risk–return trade-off. In cryptocurrency markets, where volatility is substantially higher than in traditional assets, risk-adjusted performance metrics become essential for evaluating investment attractiveness.

In this study, Sharpe, Sortino, Calmar, Rachev, Omega, Pain, and Gain/Loss ratios are used together with the Amihud illiquidity measure. Table 2 summarises the results for BTC and SUI across different periods and sampling frequencies.

BTC results show strong regime dependence. On 2025-05-13, all risk-adjusted indicators are high (e.g., Sharpe 12.639, Sortino 18.619), indicating exceptional performance and high excess return relative to risk. In contrast, on 2024-10-01, all major metrics are strongly negative, suggesting poor investment conditions and significant downside risk.

Table 2: BTC and SUI risk–return indicators across different periods and frequencies.

	BTC 2025-05-13		BTC 2024-10-01		SUI 2024-10-01	
Metric	1 min	1 s	1 min	1 s	1 min	1 s
Sharpe ratio	12.639	1.871	-25.582	-3.564	-1.328	-0.167
Rachev ratio (5%)	1.018	0.958	0.871	0.993	0.997	1.027
Sortino ratio	18.619	1.684	-32.488	-3.498	-1.916	-0.201
Calmar ratio	408.559	6.136	-252.306	-3.981	-19.782	-0.257
Omega ratio	1.0491	1.013	0.900	0.978	0.995	0.999
Pain ratio	1451.977	21.654	-856.343	-13.810	-55.932	-0.741
Gain/Loss ratio	1.055	0.972	0.945494	0.989	0.985	1.022
Average Amihud illiquidity	0.0000000000	0.0000000004	0.0000000001	0.00002779	0.0000000020	0.00000039

SUI exhibits consistently weaker performance. Risk-adjusted metrics are negative or close to zero, although slightly improving at higher frequency (1-second data). Liquidity remains lower than BTC, as reflected by higher Amihud values. Overall, results confirm that risk-adjusted performance in cryptocurrency markets is highly time-dependent and sensitive to sampling frequency.

A full list of 40 used indicators includes: (i) alpha-stable parameters after removing zero returns ($\alpha_0, \beta_0, \sigma_0, \mu_0, N_0$); (ii) alpha-stable parameters for the full dataset ($\alpha_a, \beta_a, \sigma_a, \mu_a, N_a$); (iii) empirical indicators (mean, standard deviation, skewness, excess kurtosis, median, MAD, quartiles, selected percentiles); (iv) risk measures (semi-variance, downside deviation, VaR/CVaR 95%, tail risk, max drawdown, entropy-based risk, annualized volatility); and (v) performance measures (Sharpe, Sortino, Calmar, autocorrelation (lag 1), Rachev (5%), Omega, information ratio, Pain, Gain/Loss, Roll's spread estimate, Amihud illiquidity). All these indicators can be used to train machine learning algorithms and neural networks.

Conclusions

This study analysed high-frequency cryptocurrency market data, focusing on price dynamics, liquidity, and risk–adjusted performance across different time resolutions.

The results show that cryptocurrency markets exhibit strong time-scale dependency. Statistical properties of returns, as well as risk–return indicators, differ substantially between 1-second and 1-minute data, with higher-frequency data revealing greater variability and microstructure effects.

Return distributions are clearly non-Gaussian and exhibit heavy tails. A significant proportion of zero returns is observed at ultra–high frequencies, while non-zero returns are better described by flexible distributions such as alpha-stable models.

Liquidity analysis confirms that Bitcoin is significantly more liquid than smaller assets, as reflected by tighter spreads and lower Amihud illiquidity. Spread dynamics are time-varying and increase during periods of market stress, providing useful signals about changing market conditions.

Risk-adjusted performance is highly regime-dependent. Certain periods show exceptionally strong performance, while others are characterised by negative returns and elevated downside risk. This highlights the importance of evaluating performance across different time periods and frequencies.

Overall, the findings demonstrate that robust analysis of cryptocurrency markets requires combining distributional modelling, liquidity measures, and multi-scale risk evaluation.

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Leeds Institute for Fluid Dynamics: Fluids in Processing & Safety

The Leeds Institute for Fluid Dynamics (LIFD) on behalf of the University of Leeds hosted the 191st European Study Group with Industry (ESGI) focused on applying mathematical techniques to solving industrial fluid problems. This week-long workshop welcomed early career researchers in applied mathematics, fluid dynamics, engineering and related fields to investigate critical safety related areas. Specifically, three problems related to gas transport and release were considered, as proposed by our industrial partners. Solutions were developed using both analytical and numerical techniques.



The event was held at University of Leeds (UK) from 23 to 26 June 2025.

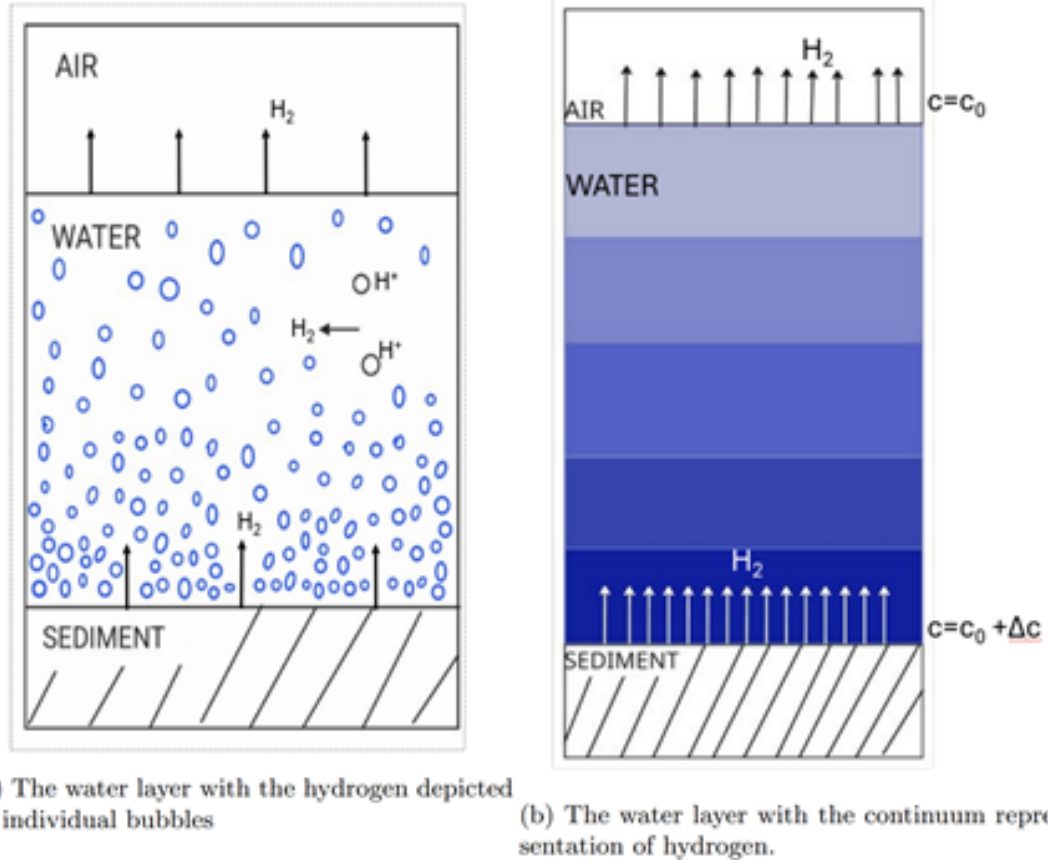
Event Summary

The event hosted >30 researchers from across the UK and Europe, with a focus on providing training in research problem solving to PhD students and Early Career Researchers (ECRs). The workshop was supported by three industrial partners with similar R&D problems in fluids safety; Sellafield Ltd (the UK's largest nuclear licence site operator), ABL Group (a global consultancy group, focused on supporting safety and sustainability in maritime, oil & gas and renewable energy sectors) and the Health and Safety Executive (HSE). Each industry partner led their own case study, with researchers equally distributed between them. Sellafield's case study focused on analysing hydrogen transport through nuclear waste storage tanks, while ABL Group also focused on hydrogen safety, in this case, developing solutions to predict the concentration spread from ruptured pipelines. Showing strong links between all case studies, HSE were similarly interested in ruptured pipelines, but here defined an

R&D problem examining the ground height spread of CO_2, during a sequestration accident scenario. Listed below are summaries of each individual case study, along with the main scientific results from the workshop.

Case Study 1: Sellafield

This case study developed a mathematical model to understand hydrogen transport within a liquid layer, for predicting release in nuclear waste storage tanks and silos at Sellafield. Hydrogen is generated through both corrosion of Magnox waste and radiolysis, posing a risk of hazardous atmospheric release during decommissioning. The system is simplified into three layers—sediment, water, and air—focusing on the water layer, where hydrogen exists as both dissolved gas and small bubbles, treated as a continuous concentration field (Figure 1). The governing equations combine advection–diffusion of hydrogen with fluid momentum and density variations, where density depends on hydrogen concentration and pressure.



(a) The water layer with the hydrogen depicted as individual bubbles

(b) The water layer with the continuum representation of hydrogen.

Fig. 1: Schematic diagrams showing the different representations of hydrogen release in Case Study 1. (a) shows the individual hydrogen bubbles within the water layer, while (b) represents the hydrogen concentration, with regions of higher concentration depicted with darker blue.

Using the Boussinesq approximation and linear stability analysis, the study investigated when disturbances cause stable behaviour or trigger convection leading to hydrogen release. Key dimensionless parameters, especially

concentration and pressure Rayleigh numbers (see Figure 2), determine whether the system is stable, exhibits oscillatory instability, or undergoes rapid “fingering” instability.

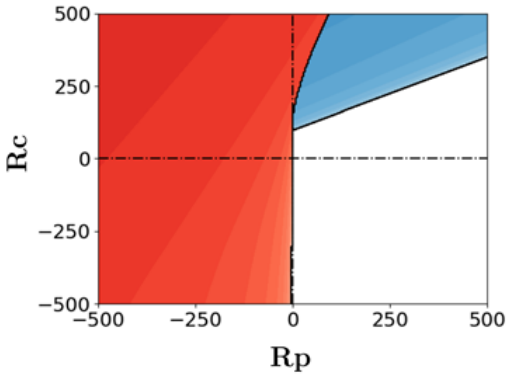


Fig. 2: Plot of the growth rate of perturbations in hydrogen release. White space denotes stability (perturbations do not grow), red shading direct instability and blue shading oscillating instability. Darker shading denotes faster growth, quantifying the amount of atmospheric hydrogen release post-disturbance. The black dotted lines denote the lines $Rp = 0$ and $Rc = 0$.

Results showed that hydrogen release requires sufficiently strong concentration gradients, with pressure gradients generally stabilising the system. In some regimes, oscillatory convection may limit release, while stronger gradients can cause dangerous rapid discharge. Overall, the model provides insight into conditions governing hydrogen release risk and supports safer nuclear waste management.

Case Study 2: ABL Group

The project developed a computationally efficient analytical approach for predicting hydrogen leak dispersion near populated or industrial areas. It applies a Gaussian plume model with assumptions such as steady-state conditions, flat terrain, and constant wind. Choked flow equations estimate leak rates, while Pasquill stability criteria define dispersion coefficients. Plume rise is evaluated using the Briggs model, with analysis showing momentum dominates over buoyancy effects. Figure 3 presents a schematic of the plume, along with Gaussian spread model framework. The model was used to predict hydrogen density relationships and spread for different pipeline pressures, helping better inform safety cases for leak scenarios.

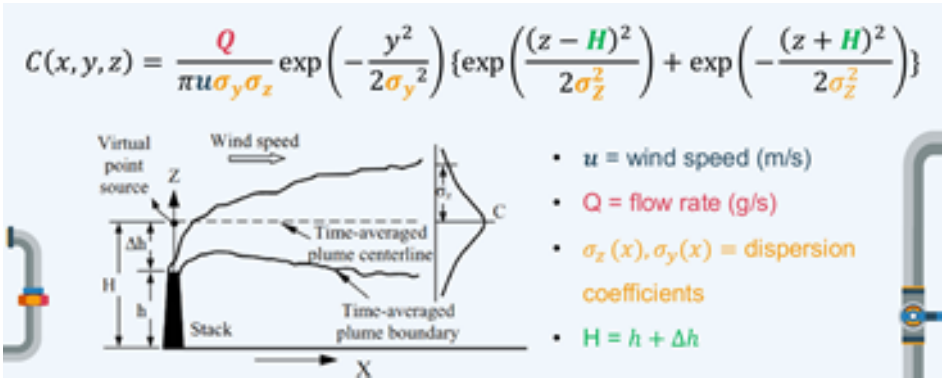


Fig. 3: Schematic of plume model with underlying Gaussian equation.

Preliminary results are presented in Figure 4, showing both vertical and lateral spread of the leak at the source height (where the dashed red line highlights the lower flammability limit of 4% H2 in air).

Over an extended period at high pipeline pressures (Figure 4 (a)) the flammability envelope extends hundreds of metres downwind.

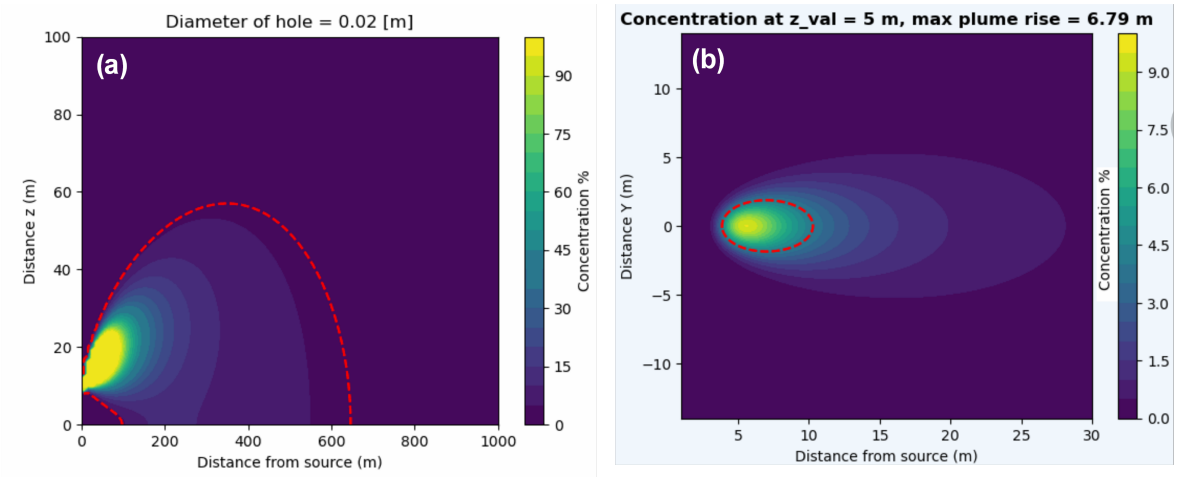


Fig. 4: Height versus length spread of high-pressure plume with rupture hole of 0.02 m (a) and lateral spread of low-pressure plume (b). Dashed red line indicates lower flammability limit.



Fig. 5: QGIS mapping of hydrogen plume lower flammability limit for three increasing pipeline pressures (214, 250, 300 bar).

Results are also visualised in QGIS, mapping flammable limits. Figure 5 shows the map of the hydrogen concentration (at the lower flammability limit) for three pipeline pressures indicating spread over the University of Leeds (with a prevailing wind left to right). Additional validations were also undertaken, comparing outputs to experimental data, with future work focusing on improved near-field modelling and CFD comparison.

Case Study 3: HSE

This study examined dense gas dispersion modelling for carbon dioxide (CO_2) leaks from pipelines, highlighting safety risks when concentrations exceed 5% (due to potential asphyxiation). The investigation was motivated by the Satartia pipeline rupture (Figure 6(a)), where terrain effects were underestimated. The study aimed to develop improved, computationally efficient models that account for sloped landscapes and incorporated wind effects (see predicted system in Figure 6(b)). Two methods were explored: a detailed CFD approach using ANSYS Fluent, and a computationally efficient shallow-layer model that includes air entrainment and gravity-driven flow. Governing equations for mass, momentum, and concentration are formulated, and numerical methods are then applied.

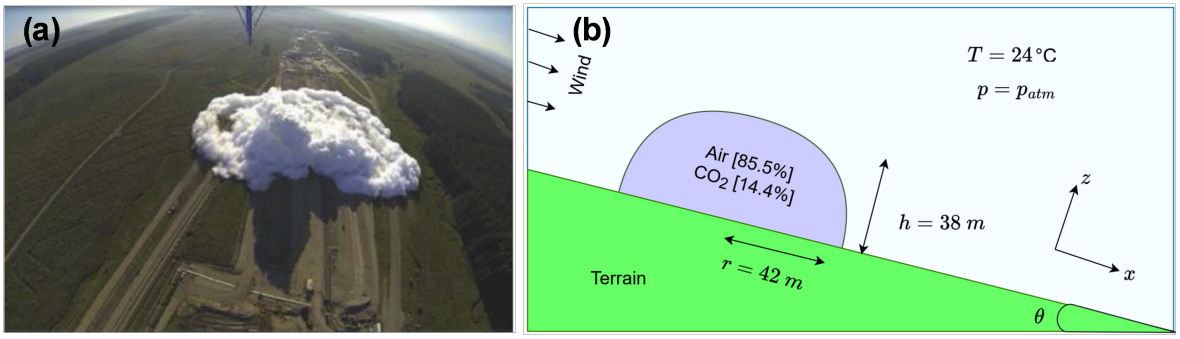


Fig. 6: (a) Image of Satoria CO_2 rupture and (b) Schematic of HSE model system setup parameters.

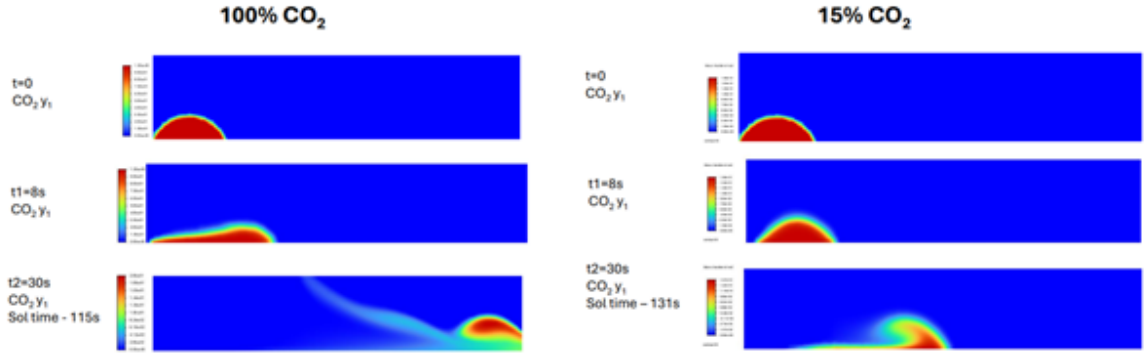


Fig. 7: Dispersion concentration changes of CO_2 gas spherical cap over time for 100% CO_2 gas (left) and 15% mixture with air (right).

Presented in Figure 7 are the Fluent CFD results, modelling turbulence using unstable RANS, assuming incompressibility. Given are cases at both 100% CO_2 and 15% CO_2 (the latter considered a more realistic pipeline gas mixture at rupture). The influence of the dense CO_2 is observed particularly with the 100% case, where the gas blob (initiated as a spherical cap) becomes flatter to the surface, and spreads faster than the low concentration 15% mixture (due to the angled terrain). Significant convective currents lead to unstable mixing and dispersion of the blob, with significantly reduced concentrations in the blob wake, due to the influence of the prevailing tail wind. Overall, this work aims to improve risk assessment and guide future large-scale validation experiments that are currently being designed by the HSE.

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9

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Mathematics, as the universal language of the sciences, plays a decisive role in technology, economics and the life sciences. European industry is increasingly dependent on mathematical expertise in both research and development to maintain its position as a world leader for high technology and to comply with the EU 2020 agenda for smart, sustainable and inclusive growth.

The European Consortium for Mathematics in Industry (ECMI) is a group of academic institutions, industrial companies and private researchers that act co-operatively to meet these needs. Specific ECMI initiatives aimed at ensuring mathematics contributes to the advancement of science and technology include:

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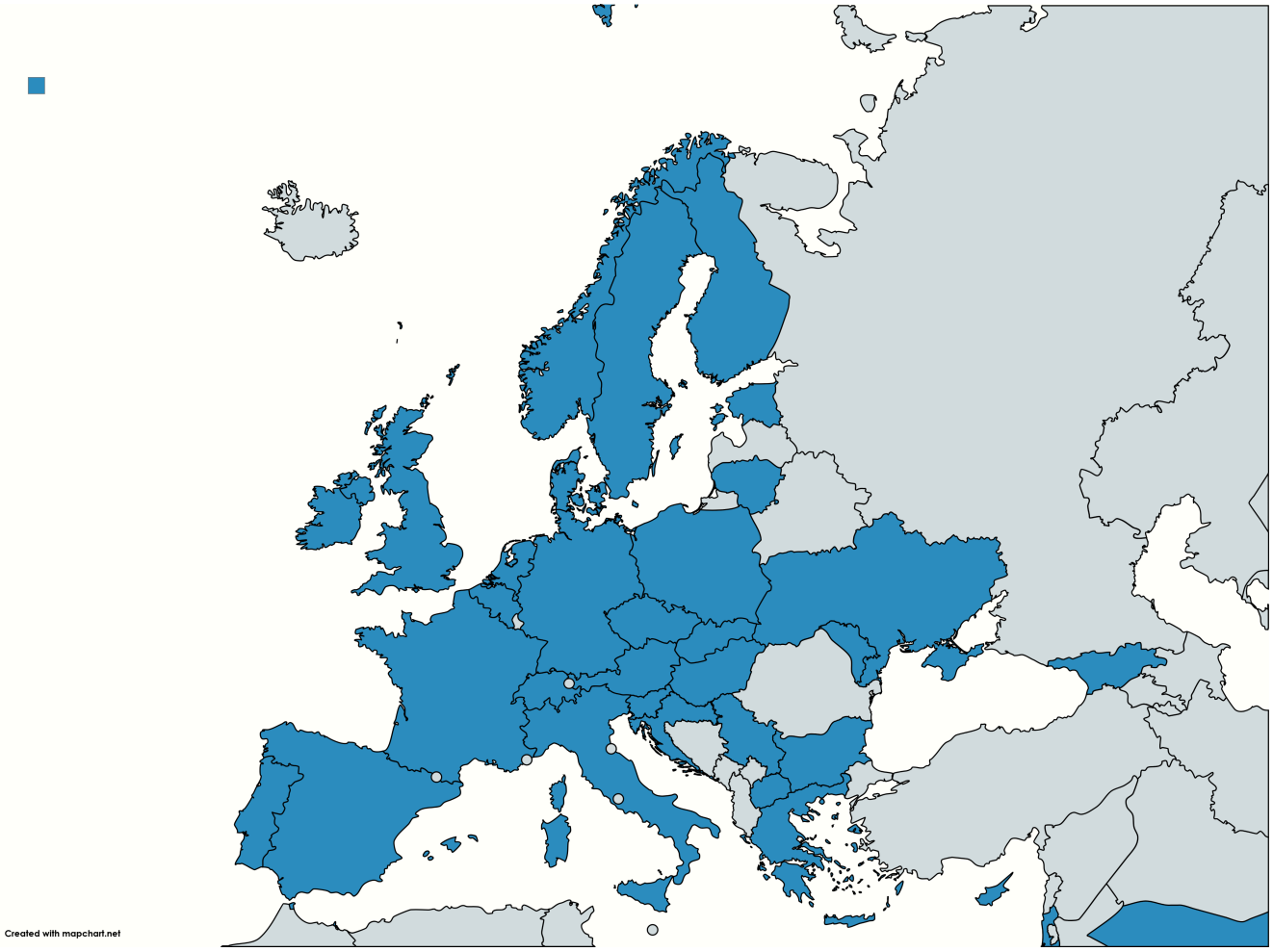
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